Capital stocks, capital services, and depreciation: an integrated framework

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Contents

Abstract

Neo-classical theory provides an integrated framework by means of which we can measure capital stocks, capital services and depreciation. In this paper the theory is set out and reviewed. The paper finds that the theory is quite robust and can deal with assets like computers that are subject to rapid obsolescence. Using the framework, estimates are presented of aggregate wealth, aggregate capital services and aggregate depreciation for the United Kingdom between 1979 Q1 and 2002 Q2, and the results are tested for sensitivity to the assumptions. We find that the principal source of uncertainty in estimating capital stocks and capital services relates to the treatment and measurement of investment in computers and software. Applying US methods for these assets to UK data has a substantial effect on the growth rate of capital services and on the ratio of depreciation to GDP.

Key words: capital stocks, capital services, depreciation

JEL classification: E22, O47

Summary

This paper presents an integrated framework to measure capital stocks, capital services, and depreciation. The framework is integrated in two senses: first, our approach to measuring each of these variables is intellectually consistent; second, we use a common set of data for all three variables. Much of the difficulty of deriving good measures of aggregate capital, whether stocks or services, derives from two basic empirical facts. First, the relative prices of different types of asset are changing. Second, the pattern of investment is shifting towards assets with shorter economic lives. So we cannot treat capital as if it were composed of a single homogeneous good. To some extent, these two facts are aspects of the same important economic change: the shift in the pattern of investment towards information, communications and technology (ICT) assets. The relative prices of these assets are falling rapidly and their economic lives are much shorter than those of most other types of plant and machinery.

Theory

The wealth concept of capital, while appropriate for some purposes, is not the right one for a production function or for a measure of capacity utilisation. For the latter purposes, we need a measure of aggregate capital *services*. A second concept of aggregate capital, which will be called here the volume index of capital services (VICS), answers this need.

In principle, the VICS measures the flow of capital services derived from all the capital assets, of all types and all ages, that exist in a sector or in the whole economy. Methodologically, the main difference between the VICS and wealth-type measures of capital is the way in which different types and ages of assets are aggregated together. In the VICS, each item of capital is (in principle) weighted by its rental price. The rental price is the (usually notional) price that the user would have to pay to hire the asset for a period. By contrast, in wealth measures of the capital stock each item is weighted by the asset price.

An important practical implication of using a VICS rather than a wealth measure is that the VICS will give more weight to assets like computers and software for which the rental price is high in relation to the asset price.

We review the theory of, and empirical evidence on, depreciation. The assumption that depreciation is geometric greatly simplifies the theory and seems consistent with the (limited) facts. We also consider whether the geometric assumption is appropriate for assets like computers. Computers do not suffer much from physical wear and tear, but nevertheless have very short lives due to what is usually called ‘obsolescence’. We find that, in principle, our framework encompasses obsolescence. Nevertheless, we show that in practice depreciation rates may be somewhat overstated owing to failure to control fully for quality change.

Empirical measures of wealth and VICS

We adopt the geometric assumption in our empirical work for the United Kingdom. Because of the uncertainty about asset lives and the pattern of depreciation in the United Kingdom, we calculate wealth and VICS measures under a range of assumptions. We test the sensitivity of our

results in three main ways. First, we compare results using both US and UK assumptions about asset lives. Second, we compare results based on a comparatively coarse breakdown of assets into four types only, with results derived from a more detailed breakdown in which computers and software are distinguished separately. Third, we compare the effect of US versus UK price indices for computers and software. Our results are for the whole economy and all fixed assets excluding dwellings, for the period 1979 Q1-2002 Q2. Our main findings for wealth and VICS are as follows:

1. Using the conventional National Accounts breakdown of assets into buildings (excluding dwellings), plant and machinery, vehicles, and intangibles, we find that the *growth rates* of wealth and the VICS are insensitive to variations in depreciation rates (ie, asset lives). In these experiments the rates for each asset are assumed constant over time.
2. However, the *level* of wealth is quite sensitive to variations in depreciation rates.
3. Still sticking with the conventional asset breakdown, wealth and VICS grew at similar rates over the period as a whole. In the 1990s, the gap between the two measures widened a bit, with the growth rate of the VICS higher by about 0.1 percentage points per quarter.
4. The effect on the estimates of separating out computers and software is quite complex. First, much larger differences appear between the growth rates of VICS and wealth, of the order of 0.2-0.4 percentage points per quarter. Second, the growth rate of wealth tends to be slower, though that of the VICS is not necessarily faster. But when we apply the set of assumptions closest to US methods, the growth rate of the VICS is raised by 0.2 percentage points per quarter, relative to the VICS with computers and software included with other asset classes.

These results suggest that the treatment and measurement of investment in computers and software is an empirically important issue. It is common ground that the relative price of these assets has been falling, so it is in principle correct to separate them out explicitly – and it matters in practice. The conclusions about the growth rates of both VICS and wealth turn out also to be sensitive to the price index used for computers and to the way in which the level of software investment is measured.

The wealth and VICS estimates under a variety of assumptions can be downloaded from the Bank of England's website (www.bankofengland.co.uk/workingpapers/capdata.xls).

The aggregate depreciation rate and the ratio of aggregate depreciation to GDP

We also estimate aggregate depreciation (capital consumption) for the same range of assumptions. We study the sensitivity of the aggregate depreciation rate and of the ratio of depreciation to GDP to the assumptions, and compare our estimates with ones derived from official data. We find:

1. Using the conventional asset breakdown and our assumptions about depreciation rates at the asset level, there is no tendency for the aggregate depreciation rate to rise over the past two decades.

2. Separating out computers and software has less effect than one might have expected: even the use of US methods raises the aggregate rate by only about 1 percentage point, to 7% in 2000, and again there is no sign of an upward trend. The reason is that, even by 2000, the share of computers and software in wealth was only about 4% in the United Kingdom. By contrast and on a comparable basis, the aggregate depreciation rate in the United States has trended smoothly upwards since 1980, to reach nearly 9% in 2000. This illustrates the much greater scale of ICT investment in the United States.
3. Assumptions about asset lives have a large impact on the estimated ratio of depreciation to GDP. The official UK National Accounts measure has been drifting down fairly steadily since 1979. In 2001 it stood at 8%. Using shorter US asset lives and the conventional asset breakdown, the ratio was over 10% in the same year. Separating out ICT assets and using US methods, the ratio rises to nearly 13%, similar to the ratio in the United States. Interestingly, in neither country was there any upward trend in the ratio, except perhaps in the past couple of years. The reason is that, although the quantity of high-depreciation assets has been growing faster than GDP, this has been offset by their falling relative price.

1 Introduction

Capital is an important part of the economy. Together with labour, it is a key factor of production, contributing to the output the economy can produce; changes in it – investment – constitute an element of demand in the economy; and it constitutes wealth, from which its owners obtain income in the form of profit.

But capital can be defined in different ways: in the context of production theory, the correct concept is the flow of capital services, whereas in the context of wealth, the correct concept is the present value of the returns accruing from the capital over its remaining productive life. And capital is difficult to measure. An economy's capital is composed of different asset types and different vintages, and both the value of, and the services provided by, those assets change over time – eventually, to the point at which the asset has no further value or productive use. It is impossible in practice to measure those characteristics directly for each asset. So empirical measurement typically relies on measuring the rate at which new assets are acquired (gross investment) and the price of those new assets, and making a range of assumptions about how the quantity and value of older assets changes over time (loosely, depreciation).

In this paper we present an integrated framework to measure capital stocks, capital services, and depreciation, and apply it to the United Kingdom, illustrating the empirical differences which flow from the alternative concepts and different assumptions which can be made. The framework is integrated in two senses: first, our approach to measuring each of these variables is intellectually consistent; second, we use a common set of data for all three variables. Our approach is broadly neo-classical, in the tradition of Hall, Jorgenson, Griliches and Hulten. In the theoretical parts of this paper, we show that this framework is more robust than it is sometimes given credit for. Much of the difficulty of deriving good measures of aggregate capital, whether stocks or services, derives from two basic empirical facts. First, the relative prices of different types of asset are changing. Second, the pattern of investment is shifting towards assets with shorter economic lives. Because of these two facts, we cannot treat capital as if it were composed of a single homogeneous good. To some extent, though not entirely, these two facts are really aspects of the same important economic change: the shift in the pattern of investment towards information and communications technology (ICT) assets. The relative prices of these assets (at least on some measures) are falling rapidly and their economic lives are much shorter than those of most other types of plant and machinery.

Capital wealth and capital services

In current prices, the wealth represented by capital is just the sum of the values of the various asset stocks. Each stock is the cumulated sum of past investment, less the cumulated sum of depreciation (inclusive of retirement and scrapping), all revalued to current prices. In constant prices, the growth of wealth is a weighted average of the growth rates of the asset stocks, where the weights are the base-period shares of each asset in the value of wealth. Since the value of each asset is its price times its quantity, we refer to these kinds of weights as asset price weights.

Theory suggests that the wealth concept of capital, which we call for short the wealth stock or just wealth, is not the right one for a production function or for a measure of capacity utilisation.

For the latter purposes, we need a measure of aggregate capital *services*. A second concept of aggregate capital, which will be called here the volume index of capital services (VICS), answers this need.(1)

In principle, the VICS measures the flow of capital services derived from all capital assets, of all types and all ages, that exist in a sector or in the whole economy. Methodologically, the main difference between the VICS and wealth-type measures of capital is the way in which different types and ages of assets are aggregated together. In the VICS, each item of capital is (in principle) weighted by its rental price. The rental price is the (usually notional) price that the user would have to pay to hire the asset for a period. By contrast, in wealth measures of the capital stock each item is weighted by the asset price. The two types of price are of course related: the price of an asset should equal the discounted present value of its expected future rental prices.

An important practical implication of using a VICS rather than a wealth measure is that the VICS will give more weight to assets for which the rental price is high in relation to the asset price. The rental price to asset price ratio is high when depreciation is high, due to a short service life, or when the asset price is falling, so that holding the asset incurs a capital loss. If the stocks of such assets are growing more rapidly than those of other types, then the VICS will be growing more rapidly than the wealth stock. This is likely to be particularly the case at the moment, with the increasing importance of computers and similar high-tech assets that are characterised by rapid depreciation and falling prices.

Previous studies

The wealth measure of the capital stock is the more firmly established and is the standard measure produced by national statistical authorities, including the Office for National Statistics (ONS) in the United Kingdom. Statistical agencies commonly estimate two different measures of the aggregate capital stock, known generally as the gross stock and the net stock. Several different asset types may be distinguished, eg buildings, plant and machinery, vehicles, etc. Conceptually, the gross stock of any asset is simply the sum of the past history of gross investment in that asset in constant prices, less the sum of past retirements. The aggregate gross stock is just the sum of the gross stocks of the different assets. The net stock differs from the gross stock in that allowance is also made for depreciation, often at a straight-line rate over each asset's known or assumed service life.

In estimating stocks, statistical agencies nearly always employ what is called the perpetual inventory method (PIM). This starts with estimates of investment by asset and by industry or by sector. Capital stocks are then calculated by cumulating the flows of investment and subtracting estimated depreciation and retirements. Depreciation is not generally known directly, but is calculated by applying estimates of depreciation rates to the stocks. Depreciation rates may be based on asset lives (the straight-line method) or they may be deduced from econometric studies of new and second-hand asset prices (of which the best known are Hulten and Wykoff (1981a)

(1) The OECD capital stock manual (OECD 2001b) uses the term 'volume index of capital services', from which we have coined the acronym VICS. The VICS is often called the productive capital stock (by contrast with the wealth stock), but this term is highly misleading since it not a stock at all but a flow.

and (1981b)). Retirements are also not observed directly but can be calculated from estimates of the service lives of assets. Asset lives are usually derived from tax records and from surveys.[(2)]

Although the wealth concept is better known, the VICS concept is not new: it came to prominence in the seminal growth accounting study of Jorgenson and Griliches (1967) and was employed in subsequent studies by Jorgenson and his various collaborators, eg Jorgenson *et al* (1987) and Jorgenson and Stiroh (2000). The theory was set out in Jorgenson (1989); a related paper is Hall and Jorgenson (1967) on the cost of capital. Recently, the OECD has published a manual on capital measurement which contains a full discussion of the various concepts including the VICS, together with advice on how to measure it in practice (OECD (2001b)).

Versions of the VICS are already produced officially for the United States by the Bureau of Labor Statistics and for Australia by the Australian Bureau of Statistics. As far as the United Kingdom is concerned, unofficial versions of the VICS have previously been estimated by Oulton and O'Mahony (1994) for 128 industries within manufacturing (for three asset types: plant & machinery, buildings and vehicles) and by O'Mahony (1999) for 25 sectors covering the whole economy (for two asset types: plant & machinery and buildings). Oulton (2001a) contains annual estimates of the aggregate VICS incorporating explicit allowance for ICT assets. Earlier work at the Bank on the VICS is summarised in Oulton (2001b). Work is also currently under way at the ONS to produce a VICS on an experimental basis.

Plan of the paper

Sections 2 and 3 constitute the theoretical part of the paper. In Section 2 we start by reviewing the relevant part of capital theory. We discuss the relationship between asset prices and rental prices and show how this can be used to illuminate the twin issues of aggregating over vintages and aggregating over asset types. We also discuss the relationship between depreciation (how asset prices change with asset age) and what we call decay, which describes how the *services* of an asset change with age. Next, the equations of the two models used for estimating the VICS on quarterly and annual data are set out. These models make use of an important simplifying assumption, namely that depreciation is geometric. We compare the index number of the wealth measure with that of the VICS.

Section 3 is devoted to the related concepts of depreciation and replacement. Replacement is what must be spent to maintain the volume of capital *services* at the existing level, while depreciation is what must be spent to maintain the value of the capital *stock* at the existing level. We discuss the relationship between these two concepts and show that replacement and depreciation are equal when depreciation is geometric. We start by considering alternative measures of the aggregate depreciation rate. There are two broad classes of measure: nominal

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(2) Three other methods of estimating capital stocks have been employed. First, it is possible to do a sample survey or even a census of capital stocks. Such a survey has recently been done for the United Kingdom but no results have as yet been published (West and Clifton-Fearnside (1999)). Second, fire insurance values have been employed (Smith (1986)). Third, stock market values have been used (Hall (2001)). None of these methods has gained general acceptance, so they will not be considered further here. Also, stock market values can only yield a wealth measure, not a VICS. In the academic literature depreciation rates have also been derived as a by product of estimating a production function (Prucha (1997)) and scrapping has been estimated from company accounts (Wadhwani and Wall (1986)).

and real. We show that the nominal measure is consistent with economic intuition, while the real measures may behave in counter intuitive ways. For example, when a chain index is used, the aggregate real rate may rise without limit. Next, we compare straight-line with geometric depreciation. Straight-line depreciation is not a very attractive assumption empirically, but the comparison is important because many statistical agencies (including the ONS) employ the straight-line assumption. We calculate the geometric rate, which is equivalent to straight-line depreciation in a steady state, for a range of values of the service life and the steady state growth rate.

Then we turn to the vexed issue of obsolescence. We discuss the appropriate measure of depreciation when assets are subject to obsolescence. We show that obsolescence makes little difference in theory, but that it does complicate the estimation of depreciation. However, an appropriately specified hedonic pricing approach can in principle deliver good estimates of the rate of depreciation.

The remainder of Section 3 reviews the evidence on the pattern of depreciation and on the length of asset lives, for the United Kingdom and the United States. We discuss the depreciation rates used by the U.S. Bureau of Economic Analysis (BEA). We find that, as measures of *economic* depreciation in the neo-classical sense, their rates may be too high. Quantitatively, the largest divergence relates to personal computers (PCs). The BEA assumes a rate of about 40% per annum, while the study on which they rely suggests a rate of about 30% per annum as a measure of economic depreciation.

Section 4 sets out our estimates for the United Kingdom. We describe our sources and methods before going on to present our estimates for the wealth stock, the VICS, and aggregate depreciation, for a range of assumptions about depreciation and service lives, and for different degrees of disaggregation by asset type. We consider the sensitivity of our estimates to our assumptions. Finally, Section 5 concludes.

2 Theory of capital measurement

This section shows how in principle wealth and VICS can be measured from data on investment flows, asset prices and depreciation rates. There are two major theoretical issues to be settled: first, how to aggregate over vintages of a given type of asset, and second, how to aggregate over different asset types. In this section, we establish first of all the relationship between rental prices and asset prices. Then we apply this relationship to resolving these two issues.(3)

Asset prices and rental prices

Consider a leasing company that buys a new machine at the end of period t-1 and rents it out during period t. It pays a price $p^A_{t-1,0}$, where the superscript 'A' indicates this is an asset price.

(3) Our treatment draws heavily on Jorgenson (1989); see also Diewert (1980). Papers that focus on depreciation include Hulten and Wykoff (1996) and Jorgenson (1996). An exhaustive discussion of the concept of the VICS, together with a summary of research in this area, empirical findings and the practices of national statistical agencies, is in the OECD manual on measuring capital (OECD (20001b)); a shorter treatment is in the OECD productivity manual (OECD (2001a)).

The first subscript indicates the time at which the asset is acquired, the second the asset's age (zero in this case, since it is new). By definition, the value of the leasing company's investment one period later, at the end of period t, is $(1+r_t)\cdot p^A_{t-1,0}$ where r_t is the actual nominal rate of return during period t (this may differ from the equilibrium rate of return). What does the return actually consist of? During period t the leasing company rents out the asset and at the end of t it is paid a rental which we write as $p^K_{t,0}$. Here the first subscript denotes the period in which the rental is received and the second the asset's age. The superscript 'K' indicates that this is the *rental* price for capital services (K), as opposed to the *asset* price (denoted by a superscript 'A'). At the end of period t, the leasing company has an asset which is now one year old and which can (if desired) be sold for a price $p^A_{t,1}$. So the value of the leasing company's investment is (ignoring tax for the moment):

$$(1+r_t)\cdot p^A_{t-1,0} = p^K_{t,0} + p^A_{t,1} \tag{1}$$

Iterating this equation forward, we obtain:

$$p^A_{t-1,0} = \sum_{z=0}^{n}\left[p^K_{t+z,z} \Big/ \prod_{\tau=0}^{z}(1+r_{t+\tau}) \right] \tag{2}$$

assuming the asset is valueless at the end of its assumed life of n periods. That is, the asset price equals the present value of the future stream of rental prices.

From the point of view of the firm to which the leasing company rents the asset, the rental price is what it must pay for the use of the machine's services for one period. A profit-maximising firm will hire machines up to the point where the rental price equals the marginal revenue product of the machine. Under perfect competition, the rental price will equal the value of the marginal product: the output price multiplied by the machine's marginal *physical* product. So under these assumptions the rental price measures the contribution of the machine to producing output.

Though financial leasing is a common arrangement for machinery, and commercial buildings are frequently rented out by their owners, it is more common still for businesses to own their capital. In this case, they can be thought of as renting the assets to themselves. But then there is no arms-length rental price to be observed. Even in the case of leased assets, it is generally easier to observe the asset price than the rental price.

It is therefore desirable to find an expression for the (usually unobserved) rental price in terms of the asset price, which can be observed more readily. Solving equation **(1)** for the rental price:

$$p^K_{t,0} = r_t \cdot p^A_{t-1,0} + (p^A_{t-1,0} - p^A_{t,1}) \tag{3}$$

The second term on the right-hand side is the gain or loss from holding the asset for one period. Sometimes this second term is called 'depreciation', but this is not the sense in which that term is used here. Two factors affect the second term: first, the asset is now one year older, and second, time has moved on one period. It is useful to take separate account of these two factors by adding and subtracting the current price of a new machine, $p^A_{t,0}$, in the right-hand side of **(3)**:

$$p_{t,0}^{K} = r_t \cdot p_{t-1,0}^{A} + (p_{t,0}^{A} - p_{t,1}^{A}) - (p_{t,0}^{A} - p_{t-1,0}^{A}) \qquad \textbf{(4)}$$

Here the two bracketed terms on the right-hand side can be interpreted as

Depreciation: $(p_{t,0}^{A} - p_{t,1}^{A})$

Capital gain/loss: $(p_{t,0}^{A} - p_{t-1,0}^{A})$

Note that depreciation is measured as the difference between the prices of a new and a one year old asset *at a point in time t*, while the capital gain/loss is measured as the change in the price of a new asset *between periods t*-1 *and t*. Putting it another way, depreciation is a cross-section concept while capital gain/loss is a time series one, as is illustrated in the following matrix of new and second-hand asset prices:

	Period	
Age	*t*-1	*t*
0	$p_{t-1,0}^{A}$	$p_{t,0}^{A}$
1	$p_{t-1,1}^{A}$	$p_{t,1}^{A}$

Reading down the columns shows depreciation, while reading across the rows traces capital gains or losses. Defined in this way, it is quite reasonable to expect that even assets like London houses depreciate. In June 2002, the price of an 80 year old, four-bedroom terrace house in Islington may have been lower than that of a 70 year old house in Islington of comparable specification, even though the owners of both houses were hoping that their values would have risen by June 2003.

If we define the rate of depreciation during period *t* as $\delta_t = (p_{t,0}^{A} - p_{t,1}^{A}) / p_{t,0}^{A}$, then equation **(4)** becomes:

$$p_{t,0}^{K} = r_t \cdot p_{t-1,0}^{A} + \delta_t \cdot p_{t,0}^{A} - (p_{t,0}^{A} - p_{t-1,0}^{A}) \qquad \textbf{(5)}$$

which is the Hall-Jorgenson formula for the cost of capital in discrete time (Hall and Jorgenson (1967)). Equation **(5)** expresses the rental price in terms of the prices of *new* assets, the rate of return, and the depreciation rate. The prices of new assets are certainly observable; indeed they *must* be observed if we are to measure investment, and hence asset stocks, in constant prices.

Since, from now on, we will be dealing only with new asset prices, it is convenient to simplify the notation by dropping the age subscripts. But we also need to recognise explicitly that assets are of many different types. Let p_{it}^{A} be the price of a new asset of type *i* in period *t* and let p_{it}^{K} be the corresponding rental price. Then equation **(5)** can be rewritten as:

$$p_{it}^{K} = r_t \cdot p_{i,t-1}^{A} + \delta_i \cdot p_{it}^{A} - (p_{it}^{A} - p_{i,t-1}^{A}) \qquad \textbf{(6)}$$

In moving from **(5)** to **(6)** we have introduced two substantive economic assumptions, as well as a notational change. First, we are assuming that the rate of return r_t is the same on all types of asset (we write r_t rather than r_{it}). Second, we are assuming that the rate of depreciation on a new asset of a given type does not vary over time, so that we write δ_i, not δ_{it}. The first assumption is consistent with profit maximisation. Certainly, firms would like to equalise rates of return *ex ante*. But *ex post*, things might turn out differently if they are unable to adjust the size of their holdings with equal speed for all types of asset. For example, an airline may be able to adjust its stock of computers more easily than its stock of planes. The assumption of equal rates of return might be particularly hard to maintain in a recession and perhaps too in a boom characterised by 'irrational exuberance'.

The second assumption, that depreciation rates do not vary over time, is obviously not true in general. However, it is well supported as a rule of thumb by studies of second-hand asset prices (see below, Section 3). Our second assumption is much weaker than assuming geometric depreciation. But it turns out to be very convenient to assume geometric depreciation when constructing capital stocks (see below).

Notice that, to measure the value of the marginal product of capital, we do not need to ask why asset prices are changing, we just need to measure them. Also, we do not need to take a view as to the causes of depreciation. Is it due to obsolescence or to physical decay? At this point, it does not matter.

One adjustment is needed to **(6)**, to take account of taxes on profits and subsidies to investment. This can be done by introducing a tax-adjustment factor into **(6)**:

$$p_{it}^K = T_{it}\left[r_t \cdot p_{i,t-1}^A + \delta_i \cdot p_{it}^A - (p_{it}^A - p_{i,t-1}^A)\right] \qquad \textbf{(7)}$$

Here r_t must now be interpreted as the post-tax rate of return and T_{it} is the tax-adjustment factor:

$$T_{it} = \left[\frac{1-u_t D_{it}}{1-u_t}\right]$$

where u_t is the corporation tax rate and D_{it} is the present value of depreciation allowances as a proportion of the price of assets of type i.

Aggregating over vintages[(4)]

Consider a production function where output (Y) depends on the amount of the different vintages of capital which still survive and on other inputs. For notational simplicity and without loss of generality, we assume for the moment just one type of capital and one type of labour (L). Then the production function at time t+1 can be written:

$$Y_{t+1} = f(I_t, I_{t-1}, ..., I_{t-n}; L_{t+1}) \qquad \textbf{(8)}$$

where I_{t-i} is that part of investment made i years ago that still survives and the oldest assets still surviving are assumed to be n years old. Assuming constant returns to scale, by Euler's Theorem:

(4) See Fisher (1965) for a general discussion of aggregation over vintages. Diewert and Lawrence (2000) compare straight-line, geometric and one-hoss shay patterns of depreciation and discuss how the pattern affects aggregation over vintages.

$$Y_{t+1} = f_0 \cdot I_t + f_1 \cdot I_{t-1} + \ldots + f_n \cdot I_{t-n} + f_{n+1} \cdot L_{t+1} \quad \textbf{(9)}$$

where $f_s = \partial f / \partial I_{t-s}$, is the marginal product of machines of age *s*, and $f_{n+1} = \partial f / \partial L_{t+1}$ denotes the marginal product of labour. Define the aggregate capital stock *A* as:

$$A_t = I_t + (f_1 / f_0) \cdot I_{t-1} + (f_2 / f_0) \cdot I_{t-2} + \ldots + (f_n / f_0) \cdot I_{t-n} \quad \textbf{(10)}$$

where each vintage is weighted by its marginal product relative to that of a new machine. The *services* (*K*) from this aggregate are assumed to be proportional to the stock at the end of the previous period (beginning of the current period):

$$K_{t+1} = A_t \quad \textbf{(11)}$$

where the constant of proportionality is normalised to unity. Equation **(10)** is a sensible definition of the aggregate stock since we can now rewrite **(9)** as:

$$Y_{t+1} = f_0 \cdot K_{t+1} + f_{n+1} \cdot L_{t+1} \quad \textbf{(12)}$$

In other words, the contribution of all the vintages of capital to output equals the marginal product of a new machine (f_0) times the volume of capital services, as defined in equations **(10)** and **(11)**.

Another way to look at the aggregate stock is the following. Past investments $I_t, I_{t-1}, \ldots, I_{t-n}$ are all measured in the same units.(5) So to calculate their capacity to produce output it is reasonable to add them up, after allowing for the fact that the capacity of earlier investments has decayed somewhat since installation. This is what equation **(10)** accomplishes.

Equation **(11)** seems to imply that we are assuming full utilisation of capital at all times. This is not the case. As Berndt and Fuss (1986) have shown, the degree of utilisation is under certain assumptions measured correctly by the weight attached to aggregate capital services (f_0 in equation **(12)**), rather than by adjusting the capital aggregate itself. For example, if capital is underutilised during a recession, then its marginal product will be low. But then the share of profits in total income will be low too. In fact, the profit share is pro cyclical, so variations in utilisation will be captured by movements in the share, at least to some extent.

Now define the decay factor $(1 - d_s) = f_s / f_0,\ s = 0, \ldots, n$, where d_s is the rate of decay experienced by machines *s* years old.(6) Then the aggregate capital stock is:

(5) Investment in a given asset is measured in practice as the nominal value of investment deflated by a price index. The price index (eg a producer price index) in principle corrects for any quality change, so that in real terms investment is in units of constant quality. Of course, there is some doubt as to how accurately price indices do capture quality change (Gordon (1990)).

(6) The concept of decay employed here covers both 'output decay' and 'input decay' (Feldstein and Rothschild (1974); OECD (2001b)). Output decay occurs when, with unchanged inputs, the output from a given asset declines over time, eg as a result of mechanical wear and tear. 'Input decay' occurs when maintaining output requires increasing other inputs, eg rising maintenance expenditure.

$$A_t = \sum_{s=0}^{n} (1 - d_s) I_{t-s} \quad \textbf{(13)}$$

Because rental prices measure marginal revenue products, there is a connection between them and the weights in the capital aggregate **(10)**:

$$p_{t,s}^K / p_{t,0}^K = f_s / f_o \quad \textbf{(14)}$$

A great simplification is achieved if we assume that the rate of decay is constant over time: $1 - d_s = (1-d)^s, \forall s$. Here *d* is the geometric rate of decay. Then we have:

$$f_s / f_o = (1-d)^s \quad \textbf{(15)}$$

The equation for the capital stock now takes a particularly simple form. From **(13)**:

$$A_{it} = I_{it} + (1 - d_i) A_{i,t-1} \quad \textbf{(16)}$$

where we have introduced an additional subscript *i* to indicate that this relationship applies to each of potentially many types of asset.

Depreciation and decay

What is the relationship between the rate of decay and the rate of depreciation? The former is a 'quantity' concept: the rate at which the services derivable from a capital asset decline as the asset ages. The latter is a 'price' concept: the rate at which the price of an asset declines as it ages. That these are not necessarily the same can be seen from the example of assets with a 'light bulb' or 'one-hoss shay'[(7)] pattern of service (constant over the service life and falling immediately to zero at its end). In this case, decay is zero right up to the moment of failure. But a cross section of the new and second-hand prices of this asset will show the price steadily declining with age. The reason is that, though the annual return on the asset may be unchanged, the older the asset, the fewer the years over which this return is expected to be enjoyed.

However, in the case of geometric decay, it can be shown that, though the two *concepts* are different, the two *rates* are equal:

$$d_i = \delta_i \quad \textbf{(17)}$$

In this case, and only in this case, the rate of depreciation equals the rate of decay.[(8)]

[(7)] The 'wonderful one-hoss shay' (a type of horse-drawn carriage), celebrated in a poem by Oliver Wendell Holmes that is reproduced in OECD (2001b), yielded a constant flow of services before disintegrating on its 100th birthday.

[(8)] The proof comes from noting that the asset price equals the present value of the future stream of rentals: see equation **(1)**. If decay is geometric, then from **(14)** and **(15)** the rental price of an asset of age *s* in any period is $(1-d)^S$ times the price of a new asset in the same period. It follows that the corresponding asset prices must stand in the same ratio to each other. The converse is also true: if depreciation is geometric, then so is decay. See Appendix A for proof.

Aggregating over asset types

Let us say that we have solved the problem of how to aggregate over vintages of a given type of capital, but we still need to aggregate different asset types together. Suppose the true production function is given by:

$$Y_t = f(K_{1t}, K_{2t}, ..., K_{mt}; L_t, t) \qquad \textbf{(18)}$$

where there are *m* types of asset. We wish to replace this by a simpler function containing only aggregate capital services:

$$Y_t = g(K_t, L_t, t) \qquad \textbf{(19)}$$

The question is, what is the relationship between K_t and the individual K_{it}? Taking the total logarithmic derivative with respect to time in these two functions, we obtain:

$$\begin{aligned} \hat{Y}_t &= \sum_{i=1}^{m}\left(\frac{\partial \ln Y_t}{\partial \ln K_{it}}\right)\cdot \hat{K}_{it} + \frac{\partial \ln Y_t}{\partial \ln L_t}\cdot \hat{L}_t + \frac{\partial \ln Y_t}{\partial \ln t} \\ \hat{Y}_t &= \left(\frac{\partial \ln Y_t}{\partial \ln K_t}\right)\cdot \hat{K}_t + \frac{\partial \ln Y_t}{\partial \ln L_t}\cdot \hat{L}_t + \frac{\partial \ln Y_t}{\partial \ln t} \end{aligned} \qquad \textbf{(20)}$$

where a hat (^) denotes a growth rate, eg $\hat{Y}_t = d\ln Y_t / dt$. So for consistency we must have:

$$\hat{K}_t = \sum_{i=1}^{m}\left[\left(\frac{\partial \ln Y_t}{\partial \ln K_{it}}\right)\Big/\left(\frac{\partial \ln Y_t}{\partial \ln K_t}\right)\right]\cdot \hat{K}_{it} \qquad \textbf{(21)}$$

The elasticities in **(21)** are not directly observable but, if inputs are paid the value of their marginal products, they can be equated with input shares:

$$\begin{aligned} \frac{\partial \ln Y_t}{\partial \ln K_{it}} &= \frac{p_{it}^K K_{it}}{p_t Y_t} \\ \frac{\partial \ln Y_t}{\partial \ln K_t} &= \frac{p_t^K K_t}{p_t Y_t} \end{aligned} \qquad \textbf{(22)}$$

where p_t is the output price and p_t^K is the rental price of aggregate capital (the value of the marginal product of aggregate capital), so $p_t^K K_t = \sum_{i=1}^{m} p_{it}^K K_{it}$ is aggregate profit. Consequently,

$$\hat{K}_t = \sum_{i=1}^{m} w_{it}\hat{K}_{it} \qquad \textbf{(23)}$$

where:

$$w_{it} = \frac{p_{it}^K K_{it}}{\sum_{i=1}^{m} p_{it}^K K_{it}}, \quad i = 1, \ldots, m \tag{24}$$

are the shares of each type of asset in aggregate profit. Equations **(23)** and **(24)** define the VICS in continuous time as a Divisia index. For empirical purposes, we need to define it in discrete time. The discrete time counterpart of a Divisia index is a chain index. Here we use a Törnqvist chain index:

$$\ln\left[K_t / K_{t-1}\right] = \sum_{i=1}^{m} \bar{w}_{it} \ln\left[K_{it} / K_{i,t-1}\right], \quad \bar{w}_{it} = (w_{it} + w_{i,t-1})/2 \tag{25}$$

An example

Suppose that the true production function of a competitive economy is:

$$Y_t = H_t \cdot K_{1t}^{\alpha} \cdot K_{2t}^{\beta} \cdot L_t^{1-\alpha-\beta}, \qquad H_t > 0$$

where there are two types of capital. Suppose we wish to use a capital aggregate K rather than distinguish the two types. We know that the share of profit in national income is $\alpha + \beta$, so it is natural to write

$$Y_t = H_t \cdot K_t^{\alpha+\beta} \cdot L_t^{1-\alpha-\beta}$$

as the simplified production function. So for consistency we must have

$$K_t^{\alpha+\beta} = K_{1t}^{\alpha} \cdot K_{2t}^{\beta}$$

whence:

$$\hat{K}_t = \left(\frac{\alpha}{\alpha+\beta}\right) \cdot \hat{K}_{1t} + \left(\frac{\beta}{\alpha+\beta}\right) \cdot \hat{K}_{2t}$$

Here $\alpha/(\alpha+\beta)$, $\beta/(\alpha+\beta)$ can be interpreted as the shares of aggregate profit attributable to the two types of capital. This equation shows how to construct the VICS for this economy.

From theory to measurement

To calculate capital services from a particular type of asset, we need to estimate capital stocks (equation **(11)**). To calculate capital stocks, we need a back history of investment and we need to know the rates of decay (equation **(10)**). Decay rates are related to the rental prices of assets of different ages (equation **(14)**). Rental prices are normally unobserved but are related to asset prices (equation **(7)**). To estimate rental prices from equation **(7)**, we need to know also depreciation rates and the rate of return. Having estimated capital stocks, we need rental prices again to weight together the services from different assets. Depreciation rates can in principle be found by econometric analysis of a panel of new and second-hand asset prices, following the methods of Hulten and Wykoff (1981a) and (1981b) for example (see Section 3 below). To apply this approach to all types of assets would constitute a very ambitious programme of empirical research, which has not been carried out in its full entirety anywhere in the world (see Section 3 again for more on this).

The problem of estimating wealth and VICS measures on a consistent basis can be greatly simplified (both from a theoretical and an empirical point of view) if we follow Jorgenson and his various collaborators (eg Jorgenson *et al* (1987); Jorgenson and Stiroh (2000)) and assume geometric depreciation and consequently also geometric decay. Under the geometric assumption, the equations of the model (**(7)**, **(11)**, **(16)**, **(24)** and **(25)**) simplify to the following:

$$A_{it} = I_{it} + (1-\delta_i) A_{i,t-1} \tag{26}$$

$$K_{it} = A_{i,t-1} \tag{27}$$

$$p_{it}^K = T_{it}\left[r_t \cdot p_{i,t-1}^A + \delta_i \cdot p_{it}^A - (p_{it}^A - p_{i,t-1}^A) \right] \tag{28}$$

$$\begin{aligned} &\ln\left[K_t / K_{t-1}\right] = \textstyle\sum_{i=1}^m \bar{w}_{it} \ln\left[K_{it} / K_{i,t-1}\right], \\ &\qquad \bar{w}_{it} = (w_{it} + w_{i,t-1})/2,\ \ w_{it} = \frac{p_{it}^K K_{it}}{\sum_{i=1}^m p_{it}^K K_{it}},\ \ i = 1,\dots,m \end{aligned} \tag{29}$$

Empirically, this is a considerable simplification. It is assumed that we have the investment series I_{it}, the tax adjustment factors T_{it}, and the asset prices p_{it}^A. Provided we also know the depreciation rates δ_i on each asset, we can now estimate the stocks. To calculate the rental prices we need to know the rate of return too. But we can estimate this from the fact that observed, aggregate profits (Π), that is, gross operating surplus before corporation tax and depreciation, must equal the total rentals generated by all the assets:

$$\Pi_t = \textstyle\sum_{i=1}^m p_{it}^K K_{it} = \sum_{i=1}^m T_{it} \cdot \left[r_t \cdot p_{i,t-1}^A + \delta_i \cdot p_{it}^A - (p_{it}^A - p_{i,t-1}^A) \right] \cdot K_{it} \tag{30}$$

This equation contains only one unknown, r_t, so we can rearrange it to solve for the unknown rate of return. Economically, this means that we are interpreting r_t as the actual, realised, post-tax rate of return. Now we can calculate the rental prices and hence the VICS.

The model just set out is reasonable as long as the period is short (say quarterly). But if applied to annual data it is subject to two criticisms. First, the first equation states that investment done in period t is not subject to depreciation until the subsequent period. This is equivalent to assuming that investment is done at the end of the period. So if a computer is in reality purchased on 1 January 2001 the model says that it only starts depreciating on 1 January 2002. Second, capital services are assumed proportional to the stock at the end of the previous period. So a computer purchased on 1 January 2001 yields no services till 1 January 2002. Both these features are unrealistic. A slightly more complex model, which assumes that investment is spread evenly over the year and capital services are proportional to the stock at the midpoint of the year, is more appropriate for annual data. The equations of this model are as follows:

$$B_{it} = I_{it} + (1-\delta_i)\cdot B_{i,t-1}, \qquad i = 1,\ldots,m \tag{31}$$

$$A_{it} = (1-\delta_i/2)\cdot B_{it} \tag{32}$$

$$K_{it} = \overline{A}_{it} = \left[A_{i,t-1}\cdot A_{it}\right]^{1/2}, \qquad i = 1,\ldots,m \tag{33}$$

$$p_{it}^K = T_{it}\left[r_t \cdot p_{i,t-1}^A + \delta_i \cdot p_{it}^A - (p_{it}^A - p_{i,t-1}^A)\right], \qquad i = 1,\ldots,m \tag{34}$$

$$\Pi_t = \sum_{i=1}^m p_{it}^K K_{it} = \sum_{i=1}^m T_{it}\cdot\left[r_t \cdot p_{i,t-1}^A + \delta_i \cdot p_{it}^A - (p_{it}^A - p_{i,t-1}^A)\right]\cdot K_{it} \tag{35}$$

$$\begin{aligned} &\ln\left[K_t / K_{t-1}\right] = \sum_{i=1}^m \overline{w}_{it} \ln\left[K_{it}/K_{i,t-1}\right], \\ &\qquad \overline{w}_{it} = (w_{it} + w_{i,t-1})/2,\ w_{it} = \frac{p_{it}^K K_{it}}{\sum_{i=1}^m p_{it}^K K_{it}},\ \ i = 1,\ldots,m \end{aligned} \tag{36}$$

$$\begin{aligned} &\ln\left[\overline{A}_t / \overline{A}_{t-1}\right] = \sum_{i=1}^m \overline{v}_{it} \ln\left[\overline{A}_{it}/\overline{A}_{i,t-1}\right], \\ &\qquad \overline{v}_{it} = (v_{it} + v_{i,t-1})/2,\ v_{it} = \frac{p_{it}^A A_{it}}{\sum_{i=1}^m p_{it}^A A_{it}},\ \ i = 1,\ldots,m \end{aligned} \tag{37}$$

where:

m is the number of assets
A_{it} is the real stock of the ith type of asset at the *end* of period t
$\overline{A}_{it}$ is the real stock of the ith type of asset in the *middle* of period t
B_{it} is the real stock of the ith type of asset at the *end* of period t, if investment were assumed to be done at the end of the period, instead of being spread evenly through the period
K_{it} is real capital services from assets of type i during period t
I_{it} is real gross investment in assets of type i during period t
δ_i is the geometric rate of depreciation on assets of type i
r_t is the nominal post-tax rate of return on capital during period t
T_{it} is the tax-adjustment factor in the Hall-Jorgenson cost of capital formula
p_{it}^K is the rental price of new assets of type i, payable at the end of period t
p_{it}^A is the corresponding asset price at the end of period t
Π_t is aggregate profit (= nominal aggregate capital services) in period t
K_t is real aggregate capital services during period t
A_t is aggregate real wealth at the end of period t
$\overline{A}_t$ is aggregate real wealth in the middle of period t

Equations **(31)** and **(32)** describe the evolution of asset stocks. They can be shown to arise from the following accumulation equation:

$$A_{it} = (1-\delta_i/2)\cdot I_{it} + (1-\delta_i/2)\cdot(1-\delta_i)\cdot I_{i,t-1} + (1-\delta_i/2)\cdot(1-\delta_i)^2\cdot I_{i,t-2}^2 + \ldots \tag{38}$$

The factor $(1-\delta_i/2)$ arises as investment is assumed to be spread evenly throughout the unit period, so on average it attracts depreciation at a rate equal to half the per-period rate. This assumption affects the level, but not the growth rate, of the capital stock.(9)

Equation **(33)** states that capital services *during* period *t* derive from assets in place in the *middle* of period *t*. The capital stock in the middle of period *t* is estimated as the geometric mean of the stocks at the beginning and end of the period. Equation **(34)** defines the rental price of assets of type *i*. Equation **(35)** says that aggregate profits are equal to the sum over all assets of the rental price times the asset stock. Equation **(36)** defines the growth rate of the VICS and equation **(37)** the growth rate of the wealth measure.

Equations **(36)** and **(37)** are chain indices of the Törnqvist type. It would also be possible to derive growth rates of the VICS and of real wealth using fixed weights, eg those of 1995, as currently in the National Accounts. Note, however, that the ONS is planning to move to annual chain-linking in 2003.

In our empirical work, we use both models. The quarterly model uses equations **(26)-(30)**, the annual model equations **(28)-(34)**. However, at a quarterly frequency we find the estimated rental prices to be unrealistically volatile. So we use the annual model to estimate the rental prices and we employ these for quarterly, as well as for annual data.

These models assume constant rates of depreciation over time. In our empirical work we deviate from this in one respect, since we have made an allowance for accelerated scrapping during recessions. We describe this more fully in Section 4.

Wealth measures of capital versus the VICS

How does the growth of a VICS compare with the growth of a wealth measure of capital? We answer this question using the simpler quarterly model of the previous subsection. Assuming geometric depreciation, the nominal value of capital (*W*) in a balance sheet sense at the beginning of period *t* (end of period *t*-1) is:

$$W_{t-1} = \sum_{i=1}^{m} p_{i,t-1}^{A} A_{i,t-1} = \sum_{i=1}^{m} p_{i,t-1}^{A} K_{it}$$

We can define a Törnqvist index of the growth of the aggregate real stock of capital (*A*) in the wealth sense as:(10)

$$\ln\left[A_{t-1}/A_{t-2}\right] = (1/2)\sum_{i=1}^{m}\left(v_{i,t-1}+v_{i,t-2}\right)\cdot\ln\left[A_{i,t-1}/A_{i,t-2}\right]$$

$$= (1/2)\sum_{i=1}^{m}\left(v_{i,t-1}+v_{i,t-2}\right)\cdot\ln\left[K_{it}/K_{i,t-1}\right]$$

(9) This assumption corresponds to the practice of the BEA: see U.S. Department of Commerce (1999, box on page M-5).

(10) A similar index of the wealth stock is published by the BEA (Herman (2000)). Their index is Fisher rather than Törnqvist but in practice these two types of chain index yield very similar results.

where the v_{it} are the shares of each asset in the nominal value of the capital stock (V):

$$v_{i,t-1} = p^A_{i,t-1} A_{i,t-1} / \sum_{i=1}^{m} p^A_{i,t-1} A_{i,t-1}$$

The growth rate of the VICS (see equation **(29)**) is:

$$\ln[K_t / K_{t-1}] = (1/2) \sum_{i=1}^{m} (w_{it} + w_{i,t-1}) \cdot \ln[K_{it} / K_{i,t-1}]$$

The only difference between the growth rates of wealth and the VICS is the weights, $v_{i,t-1}$ instead of w_{it}. The wealth measure uses asset prices in the weights while the VICS uses rental prices, these prices being related by equation **(28)**. It is clear then that the higher the ratio of the rental to the asset price, the larger the weight that an asset will receive in the VICS. Intuitively, it is clear that if an asset has a higher-than-average rental price in proportion to its asset price, then its VICS weight will be higher than its wealth weight. This is proved formally in Appendix A.

If it turns out that the stocks of those assets with high rental price to asset price ratios tend to grow more rapidly, then a VICS will grow more rapidly than a wealth measure. Empirically, this has indeed been the case in recent decades. The service life of plant and machinery is short relative to that of buildings, hence their rental price is relative higher. And stocks of plant and machinery have grown more rapidly than those of buildings. The difference between asset and rental price weights is particularly large for assets like computers. Not only is their service life very short but their prices have been falling, ie holding them incurs a capital loss. So their rental price has to be very high (around 60% of the asset price) to make them profitable. Within the plant and machinery category, stocks of computers have been growing exceptionally rapidly (Oulton (2001a)).

In addition to the growth rates, we can if desired also derive the levels of real wealth and the VICS. In the case of real wealth, we can take the level of *nominal* wealth in some base period s (eg 1995):

$$\sum_{i=1}^{m} p^A_{is} A_{is}$$

and generate a series in 'chained 1995 pounds' by applying to this expression the growth rates given by equation **(37).** Note though that this will *not* yield the same result as would come from calculating the stock of each asset in period s prices and then adding the individual stocks. The reason is that the components of a chain index do not in general add to the chained total. Similarly, we can generate a series for the real level of the VICS by applying the growth rates given by equation **(36)** to the nominal level in base period s, which is just the level of profits in that period, Π_s: recall that the VICS measures the flow of capital services. Note that if we now compare the level of the VICS with the level of wealth, we are comparing a flow with a stock. This may be legitimate, but care should be taken over the interpretation: comparisons between the absolute size of the two measures are not meaningful.

3 Depreciation and replacement

The concepts of depreciation and replacement are related but distinct. Depreciation relates to the wealth measure of capital, replacement to the (misnamed) 'productive' capital stock, otherwise (and better) known as the VICS. Aggregate depreciation is the fall in the value of the capital stock which would occur if gross investment were zero. Alternatively, it is the amount of investment necessary to maintain the value of the stock at its current level. Replacement is the amount of investment necessary to maintain *the flow of capital services* at its current level. The difference between the two concepts is clearest in the case where the productive capacity of an asset follows the 'light bulb' pattern, ie constant up till the moment of failure. In this case, the asset falls in value with age, since there are progressively fewer years over which profits can be earned. But replacement is zero up till the moment of failure. Suppose the asset in question lasts for ten years and all investment has taken place in the last eight years. Then in the current year replacement is zero, since at the end of the year the oldest asset will be nine years old and will still be yielding the same flow of service as it did when new. But total depreciation will be positive since the assets are approaching the end of their lives. The discounted flow of future profits is falling as the assets age, so their value is declining even though their productive efficiency is unchanged.

Depreciation is also called capital consumption by national income statisticians. If depreciation is subtracted from gross investment, the result is usually called net investment. But if depreciation and replacement are not the same, then net investment so defined does not equal the increase in the VICS. There is one case, however, where aggregate depreciation and aggregate replacement are equal in value, namely when depreciation is geometric.

Consider a single, homogeneous asset, so that for the moment we ignore issues of quality adjustment. Measured in current prices, the value of the wealth stock of this asset at the end of period *t*, W_t, is:

$$W_t = p^A_{t,0}\phi_{t,0}I_t + p^A_{t,1}\phi_{t,1}I_{t-1} + p^A_{t,2}\phi_{t,2}I_{t-2} + \dots \qquad \textbf{(39)}$$

where:

$p^A_{t,s}$ is the price at time *t* of an asset which is aged *s* at *t*

$\phi_{t,s}$ is the proportion of assets of age *s* which survive at time *t* and we set $\phi_{t,0} = 1$

and I_{t-s} is the volume of investment in this asset which was carried out in period $t-s$ (the number of machines installed in *t-s*)

In period *t* prices, the value of wealth in the *previous* period, *t*-1, is:

$$W_{t-1} = p^A_{t,0}\phi_{t-1,0}I_{t-1} + p^A_{t,1}\phi_{t-1,1}I_{t-2} + p^A_{t,2}\phi_{t-1,2}I_{t-3} + \dots$$

The relationship between wealth today and wealth yesterday, measured in *today's* prices, is:

$$W_t = p^A_{t,0}I_t - D_t + W_{t-1}$$

where D is depreciation in period t prices. Hence:

$$D_t = p^A_{t,0}[(\phi_{t-1,0} - (p^A_{t,1} / p^A_{t,0})\phi_{t,1}]I_{t-1} + [(p^A_{t,1} / p^A_{t,0})\phi_{t-1,1} - (p^A_{t,2} / p^A_{t,0})\phi_{t,2}]I_{t-2} + ... \quad \textbf{(40)}$$

Note that the prices here are all dated to period t: they are the new and second-hand prices of this asset at time t, weighted by the probability of survival.

Now consider replacement investment for this asset. Assume that capital services during time t derive from assets installed in period t-1 and earlier. An asset installed in period t-s yields a flow of services during t which we denote by $p^K_{t,s}$. As in the previous section, we can think of $p^K_{t,s}$ as the *rental* price of an asset of this vintage. Define K_t as the value of total capital services from the stock of this asset at time t in period t prices. Then we have:

$$K_{t+1} = p^K_{t,0}\phi_{t,0}I_t + p^K_{t,1}\phi_{t,1}I_{t-1} + ...$$

The services that require replacement investment during t, R'_t, measured again in the prices of period t, are defined implicitly by:

$$K_{t+1} = p^K_{t,0}I_t - R'_t + K_t$$

whence:

$$R'_t = p^K_{t,0}[(\phi_{t-1,0} - (p^K_{t,1} / p^K_{t,0})\phi_{t,1}]I_{t-1} + [(p^K_{t,1} / p^K_{t,0})\phi_{t-1,1} - (p^K_{t,2} / p^K_{t,0})\phi_{t,2}]I_{t-2} + ...$$

This gives the value of the *services* that need require to be replaced: this is the reduction in the value of services which would occur if gross investment were zero. The value of *investment* needed for replacement is therefore:

$$\begin{aligned} R_t &= R'_t(p^A_{t,0} / p^K_{t,0}) \\ &= p^A_{t,0}[(\phi_{t-1,0} - (p^K_{t,1} / p^K_{t,0})\phi_{t,1}]I_{t-1} + [(p^K_{t,1} / p^K_{t,0})\phi_{t-1,1} - (p^K_{t,2} / p^K_{t,0})\phi_{t,2}]I_{t-2} + ... \end{aligned} \quad \textbf{(41)}$$

Comparing the equations for depreciation and replacement, **(40)** and **(41)**, we see that there is no reason in general to expect the two measures to be the same. The one involves asset prices, the other rental prices. However, if depreciation is geometric, then we can show that they will in fact be equal. With geometric depreciation, both the asset price and the rental price decline with age at the rate of depreciation (δ):

$$p^A_{t,s} / p^A_{t,0} = p^K_{t,s} / p^K_{t,0} = (1-\delta)^s, \quad s = 0,1,2,...$$

Hence in this case we see from **(40)** and **(41)** that:

$$R_t = D_t \quad \textbf{(42)}$$

Also the asset accumulation equation in current prices **(39)** now takes the simple form:

$$W_t = p^A_{t,0}I_t + (1-\delta)W_{t-1}$$

The aggregate depreciation rate

There is frequent interest in the aggregate depreciation rate. A rising aggregate rate suggests that the mix of assets in the capital stock is shifting towards assets with shorter lives. In turn, this implies that a given amount of gross investment will lead to lower growth in both the wealth stock and the VICS than if the mix were not changing. But we can measure the aggregate depreciation rate in either nominal terms or real terms and these measures may behave in quite different ways. In nominal terms, the aggregate rate is the ratio of aggregate nominal depreciation to aggregate nominal wealth (the nominal capital stock). In real terms, there are two measures. Either we can take the ratio of aggregate real depreciation to aggregate real wealth or we can back out the depreciation rate from the aggregate capital accumulation equation. In this subsection we show that the nominal definition has a natural interpretation. But both the real definitions can produce counterintuitive results. For example, under chain-linking the aggregate real rate can rise without limit so that it eventually exceeds the rate on any individual asset. Therefore we should exercise caution when using the real definitions.

In nominal terms, the aggregate depreciation rate is the ratio of aggregate nominal depreciation to aggregate nominal wealth (the nominal capital stock):

$$\delta_t^N = \frac{\sum_{i=1}^{m} \delta_i p_{it} A_{i,t-1}}{\sum_{i=1}^{m} p_{it} A_{i,t-1}} \qquad \textbf{(43)}$$

where δ_i is the depreciation rate on the ith asset, A_{it} is the stock of the ith asset at the *end* of period *t*, and p_{it} is the corresponding asset price.(11) One definition of the real rate is the ratio of aggregate *real* depreciation to aggregate *real* wealth:

$$\delta_t^R = D_t / A_{t-1} \qquad \textbf{(44)}$$

where D_t is aggregate real depreciation and A_t is the aggregate real capital stock at the end of period *t*.

A second definition of the real rate is the rate that can be backed out from the aggregate capital accumulation equation:

$$A_t = I_t + (1 - \delta_t^B) A_{t-1}$$

whence:

$$\delta_t^B = (I_t / A_{t-1}) + [(A_t - A_{t-1}) / A_{t-1}] \qquad \textbf{(45)}$$

where I_t is aggregate real gross investment.

(11) This definition assumes geometric depreciation, which is used below. But the nominal definition could of course be extended to the non-geometric case.

If these three measures were calculated for a single asset, the results would be identical. But when done at the aggregate level the results will differ.

Comparing the measures

(a) The nominal measure, δ_t^N

One reason for considering the nominal measure is that it squares well with the way depreciation is estimated by the BEA. In the US national accounts, the stock of any asset is assumed to evolve (approximately) according to the simple accumulation equation:

$$A_{it} = I_{it} + (1-\delta_i) \cdot A_{i,t-1}$$

where A_{it} is the stock of the ith asset at the *end* of period *t*, I_{it} is gross investment in period *t* and δ_i is the depreciation rate. With a few exceptions, the individual δ_i are not assumed to change over time.(12) So any change in the ratio of aggregate depreciation to the aggregate capital stock (in current prices) indicates a change in the asset composition of the capital stock. That is,

$$\delta_t^N = \frac{\sum_{i=1}^m \delta_i p_{it} A_{i,t-1}}{\sum_{i=1}^m p_{it} A_{i,t-1}} = \sum_{i=1}^m \left(\frac{p_{it} A_{i,t-1}}{\sum_{i=1}^m p_{it} A_{i,t-1}} \right) \cdot \delta_i$$

In other words, the aggregate depreciation rate is a weighted average of the rates on individual assets, where the weights are the shares of each asset in aggregate wealth. So the aggregate rate must necessarily be bounded by the rates on the individual assets.

(b) The first real measure, δ_t^R

Let us compare the nominal measure with the first of the two real measures, δ_t^R. We will show that, under chain-linking, the latter can produce unacceptable results. Consider a simple case where there are two assets, one with a high depreciation rate, the other with a low one. Assume that the real stock of the high depreciation asset is growing more rapidly than that of the low depreciation one (which is the case at the moment for computers and software). Suppose that the share of each asset in the value of wealth is constant over time (the Cobb-Douglas case). If the importance of the assets, as measured by wealth shares, is not changing, and the individual depreciation rates are constant, then it seems reasonable that the aggregate rate should be constant too. And this will certainly be true of the aggregate depreciation rate in nominal terms. However, it can be shown that the aggregate rate defined in real terms will rise without limit in this case. Eventually, it will be higher than either of the two individual rates! This is not a reasonable way for a measure of the aggregate rate to behave. So though we might go on using such a measure for modelling purposes, we cannot expect it necessarily to behave in ways consistent with our economic intuition.

The intuition behind this result is as follows. The growth rate of real depreciation, like the growth rate of real wealth, is a weighted average of the growth rates of the components. It can be shown that the weight on the high-depreciation asset in the depreciation index is larger than its weight in the wealth index. Consequently, if the high-depreciation asset is growing more rapidly, then real

(12) See Fraumeni (1997) and U.S. Department of Commerce (1999).

depreciation will grow more rapidly than real wealth. It follows that the ratio of real depreciation to real wealth will rise over time without limit, even if all individual depreciation rates and the shares of each asset in total wealth are constant over time (see Appendix A for a proof).

This result holds under chain-linking. With a fixed-base index, the aggregate real rate will approach the higher of the two individual rates asymptotically (see the Appendix again). In the US National Income and Product Accounts (NIPA), the growth of real depreciation, like the growth of the wealth stock, is calculated as a chain index (annual chain-linking). So this result certainly applies to the United States. In the United Kingdom, the weights are updated about every five years ('quinquennial chain-linking'). So over long periods, but not short ones, the result applies to the United Kingdom too.

(c) The second real measure, δ_t^B

Whelan (2000b) has proved a related but different result about the second real measure. Suppose we calculate the aggregate depreciation rate by backing it out from the aggregate accumulation equation:

$$\delta_t^B = (I_t / A_{t-1}) - [(A_t - A_{t-1}) / A_{t-1}]$$

which is equation **(45)**. Suppose that there are again two assets but now with the same depreciation rate. Assume that asset 1 is growing more rapidly than asset 2 but that wealth shares are constant. Clearly in this situation the aggregate depreciation rate is constant. But Whelan shows that the backed out rate δ_t^B derived from aggregate data will rise without limit. The explanation is that the weight of asset 1 in investment is higher than its weight in wealth, so investment is growing more rapidly than wealth and the I_t / K_{t-1} ratio is trending upwards.

Conclusion on aggregate depreciation measures

Measuring the aggregate depreciation rate in real terms, by either method, can lead to serious problems. So measured, the aggregate rate can be higher (or lower) than any of the rates on individual assets. And the aggregate rate can trend upwards (or downwards) without limit even though nothing is really happening in economic terms. There are certainly signs in the US data that this is not just a theoretical possibility. The upward drift in the aggregate rate is much less in nominal than in real terms. This suggests that we should use real definitions with caution. We cannot expect them necessarily to behave in ways that are consistent with our economic intuition.

Straight-line as an alternative to geometric depreciation

In the US NIPA, depreciation is assumed to be (in most cases) geometric (Fraumeni (1997)). In the United Kingdom by contrast, the ONS (along with many other national statistical agencies) assumes that assets depreciate on a straight-line basis over their assumed asset life; retirement or scrapping is assumed to be normally distributed around the mean life. Accordingly, the purpose of this subsection is to compare the implications of geometric as opposed to straight-line depreciation for the depreciation rate of a given type of asset.

The overall rate of depreciation of the stock of some asset, or the rate of deterioration of the flow of capital services which it yields, arises from two factors. First, the retirement or scrapping of assets and second, the decline in efficiency of surviving assets. We consider each of these factors in turn, first for geometric and then for straight line.

Asset mortality: geometric assumption
Suppose that in some given year a number of machines of a particular type are added to the capital stock. We refer to these machines as a cohort. Suppose that there is a fixed probability of 'death' attached to this asset type and that this probability is independent of age. 'Death' might mean loss due to accidents, fires or explosions or it might mean voluntary scrapping for any reason. Denote the probability of death by m. Then the probability that a new example of this asset lives for t years is:

$$(1-m)^t m$$

This is a geometric distribution, so the expected life n of a new asset is:

$$n = (1-m)/m \approx 1/m \text{ for small } m$$

(See eg Feller (1968), Vol. 1, page 268.) Hence the mortality rate m is related to the mean life as:

$$m = 1/(n+1)$$

We can also ask: what proportion of the original cohort is expected to survive after L years? This is given by:

$$(1-m)^L = [1-1/(n+1)]^L$$

For n = 5, 10, 20, or 30 we get 40%, 39%, 38% and 37% respectively, substantially less than 50%. This is not surprising. The period of time after which only 50% of the original cohort survives is the *median* life. With a distribution skewed to the right, as is this one, the median life is less than the mean life. In fact for this distribution, the median life is about 70% of the mean life:

Table A Mean and median life lengths under geometric depreciation (years)

Mean	*Median*
5	3.8
10	7.3
20	14.2
30	21.1
40	28.1

Note: The median life n is calculated as the solution to $[1-(1/n+1)]^n = 0.5$ ie $n = \ln(0.5)/\ln[1-1/(n+1)]$.

Declining efficiency with age
According to basic capital theory, the price of an asset is the present value of the services it is expected to yield over its remaining life (see Section 2 above). 'Services' refers to the marginal

revenue product of the asset. If assets could be hired, then the rental price would equal the asset's marginal revenue product, in the same way that the wage equals the marginal revenue product of labour. If assets are not expected to last forever, then older assets will command a lower price than newer ones at any point in time, simply because the stream of future services is expected to be shorter. This effect is already accounted for in the discussion of mortality. But in addition, there is the possibility that the services of a surviving asset decline with age. This gives an additional reason for the asset price to decline with age and also must be accounted for if we want to measure capital services correctly. For present purposes, we do not need to discuss why an asset's services might decline with age, just to examine the consequences if this is indeed occurring.

The standard way this has been dealt with in practice in the United States is to assume that depreciation is 'accelerated' by comparison with what would occur if scrapping were the only force at work. The depreciation rate is expressed as:

$$\delta = R / n$$

where R is termed the 'declining balance rate'. In the past, R = 2 was frequently chosen; this is referred to as the 'double declining balance' method. This implies that the efficiency of surviving assets declines at the constant rate $1/n$ while separately and independently the force of mortality is $1/n$ too: that is, $(1-\delta) = (1-1/n)\cdot(1-1/n) \approx 1-2/n$. In the US NIPA a variety of values of R are now employed, based on the Hulten-Wykoff studies. Typically, R = 1.65 for equipment and R = 0.91 for private non-residential structures (Fraumeni (1997)).

Straight-line depreciation

Under straight-line depreciation, the gross stock (GA) of asset i at the end of period t is the cumulated sum of all surviving vintages of investment:

$$GA_{it} = \sum_{s=0}^{n_i-1} I_{i,t-s}$$

where I_{it} is gross investment in asset i during period t and n_i is the asset's life. Under the assumption of straight-line depreciation, an asset loses a fraction $1/n_i$ of its *initial* value in each period. Since assets of each surviving vintage depreciate by an equal amount per period, overall depreciation (capital consumption) on asset i in period t, D_{it}, is:

$$D_{it} = GA_{it} / n_i$$

The net stock of asset i at the end of period t is the gross stock less cumulated depreciation:

$$A_{it} = GA_{it} - \left[\sum_{s=0}^{n_i-1} sI_{i,t-s} / L_i\right]$$

The depreciation *rate* is defined as depreciation in period t as a proportion of the *net* stock of the asset at the end of the previous period:

$$\delta_{it} = D_{it} / A_{i,t-1}$$

An important point to note is that the straight-line depreciation rate in general varies over time, even when asset life is assumed constant. The rate is in fact a function of the age structure of the stock: the younger the stock, the lower the rate. Suppose that the stock consists entirely of the oldest surviving vintage, there having been no subsequent investment. Then the gross stock is $I_{i,t-L+1}$, the net stock is $I_{i,t-n_i+1}/n_i$, and depreciation is $I_{i,t-n_i+1}/n_i$. So the depreciation rate is 1. On the other hand, suppose the gross stock consists entirely of investment done in the last period, $I_{i,t-1}$. Then depreciation is $I_{i,t-1}/n_i$ and the depreciation rate is $1/n_i$. So in general, the depreciation rate varies between $1/n_i$ and 1. If an investment boom occurs, then other things equal the depreciation rate falls.

Geometric versus straight-line depreciation
Suppose that an asset costs £1 when new in year *t*. If depreciation is geometric, then the actual nominal value of depreciation (or capital consumption) on an asset aged *s* years in year *t* is

$$\delta(1-\delta)^s$$

Clearly, capital consumption itself declines geometrically as the asset ages (*s* rises). It is highest in the asset's first year and approaches zero asymptotically as the asset ages. But by definition the depreciation *rate* (depreciation as a proportion of the asset's price) is constant.

With straight-line depreciation, capital consumption on a particular asset type is the same at each age, equal to a fraction 1/*n* of the price when new. So depreciation as a proportion of the second-hand asset price, the depreciation rate, is rising as the asset ages. The depreciation rate for an asset of age *s* is:

$$\frac{1}{n-(s-1)}$$

This equals (100/*n*)% in the first year of life and 100% in the last year of life. It follows that the total depreciation rate on a particular asset class depends on the age structure of the stock. Under geometric depreciation by contrast, total depreciation is independent of the age structure.

Because the price of an asset is the present value of the services it is expected to yield over its remaining life, there is a connection between depreciation and the rate at which services are changing (decay). If depreciation is geometric, then decay is geometric too and at the same rate. So, under geometric depreciation, old assets never apparently die but just fade away. This is best understood in a probabilistic sense: individual members of a cohort die, but the cohort as a whole goes on forever, though eventually its size is insignificant. Under straight-line depreciation, it can be shown that services decline linearly with age and then fall instantaneously to zero at the end of the asset's finite life. The main problem with straight-line depreciation is that it does not fit the facts. Empirical studies show that the age-asset price profile is generally convex, which is consistent with geometric depreciation. Under straight-line depreciation, the asset price should decline with age in a linear fashion.[13]

[13] See Oulton (2001b) for more on this.

Quantitative comparison between straight-line and geometric depreciation is not straightforward, since in the former the depreciation rate depends on the age structure of the stock. But a useful benchmark is provided for the case where investment is growing at a constant rate over the assumed life of the asset (a steady state). Table B below illustrates for a variety of asset-life lengths and growth rates of investment. The straight-line rate declines as the growth rate rises, since this shifts the age structure towards more recent vintages. But the effect is not very marked, except for the longest lives and high growth rates (which are in any case unlikely for long-lived assets). A five-year life corresponds in steady state to a geometric growth rate of about 30%-33% and a 20-year life to a geometric rate of 6%-9% (8%-9% if growth does not exceed 5% per annum).

In the United States, the depreciation rate for plant and machinery (excluding computers and software) averages about 13%. In the United Kingdom, the ONS assumes that plant and machinery has a life of 25-30 years in most industries. Table B shows that this is equivalent to a geometric rate of 5%-9% if growth does not exceed 5% per annum, much lower than the US rate.

Table B Steady-state depreciation rates when depreciation is straight line

	Growth rate of investment (% per annum)			
Life (years)	2	5	10	20
5	32.90	32.28	31.35	29.77
10	17.66	16.95	15.94	14.44
20	8.96	8.27	7.41	6.44
30	5.89	5.25	4.58	3.98
80	1.97	1.62	1.43	1.33

How do the levels of asset stocks and depreciation vary with the depreciation rate?
In a steady state, the *growth rate* of the stock of an asset is constant and equal to the growth rate of the investment which generates the stock. This is true whether depreciation is straight-line or geometric. But the depreciation rate does affect the *level* of the stock. Assuming a steady state, from the basic capital accumulation equation we can find that the end-year stock (A) is related to the investment flow by:

$$\frac{A_t}{I_t} = \left[\frac{1+g}{g+\delta}\right]$$

where g is the steady-state growth rate. The steady-state level of depreciation, as a proportion of gross investment, is therefore given by:

$$\frac{\delta A_{t-1}}{I_t} = \left[\frac{\delta}{g+\delta}\right]$$

These ratios are shown in Tables C and D. We see that the stock/gross investment ratio falls as the depreciation rate rises. The ratio is also negatively related to the growth rate. On the other hand, the depreciation/gross investment ratio rises with the depreciation rate. This is relevant when considering how the depreciation/GDP ratio might be expected to behave (though here aggregation issues and relative prices will also play a role). For example, if we change our estimate of the depreciation rate from 10% to 15%, then (for growth rates not exceeding 5%) we will lower the level of the asset stock by between 25% and 29%, while raising depreciation by 6%-12%.

Depreciation as a proportion of gross investment is high when growth rates are low and when depreciation rates are high. For computers, where the stock might grow at 20% per annum and the depreciation rate is 30%, the proportion would be 60% in steady state.

Table C Ratio of asset stock to investment in steady state

1.1.1	*Growth rate (g)*			
Depreciation rate (δ)	*0.02*	*0.05*	*0.10*	*0.20*
0.02	25.50	15.00	9.17	5.45
0.05	14.57	10.50	7.33	4.80
0.10	8.50	7.00	5.50	4.00
0.15	6.00	5.25	4.40	3.43
0.20	4.64	4.20	3.67	3.00
0.30	3.19	3.00	2.75	2.40
0.40	2.43	2.33	2.20	2.00

Table D Ratio of depreciation to investment in steady state

	Growth rate (g)			
Depreciation rate (δ)	*0.02*	*0.05*	*0.10*	*0.20*
0.02	0.50	0.29	0.17	0.09
0.05	0.71	0.50	0.33	0.20
0.10	0.83	0.67	0.50	0.33
0.15	0.88	0.75	0.60	0.43
0.20	0.91	0.80	0.67	0.50
0.30	0.94	0.86	0.75	0.60
0.40	0.95	0.89	0.80	0.67

Conclusion on straight-line versus geometric depreciation

Given the asset lives of plant and machinery assumed by the ONS, the equivalent geometric rate for the United Kingdom is substantially lower than its US counterpart. If the rates used to estimate UK asset stocks and depreciation were raised to US levels, the level of stocks would be lowered by a substantial amount, while depreciation as a proportion of gross investment and GDP would be simultaneously raised. These effects are quantified in Section 4 below.

Obsolescence and the interpretation of depreciation

Up to now we have treated the concept of depreciation, and the related concept of decay, as unproblematic in theory, even if difficult to measure in practice. But there are some important conceptual issues that need to be resolved. These revolve around the concept of obsolescence and have been made more acute by the rising importance of assets like computers. These depreciate rapidly but do not decay physically in any obvious sense. In this subsection we show that the basic framework is unaffected when assets suffer from obsolescence. But obsolescence does raise some tricky questions about how to measure depreciation. We show that these can in principle be resolved within an appropriately specified hedonic pricing approach.

Obsolescence versus physical decay

Some assets, like buildings, decay with age. Mechanical wear and tear causes many types of machinery to decay with use.(14) But some assets, in particular computers and software, suffer little or no physical decay but are nevertheless discarded after relatively brief service lives. The cause is usually said to be 'obsolescence', due to the appearance of newer and better models. Does this make any difference to the analysis above?

The answer is no. The weights in equation **(10)** are relative marginal products. Certainly these may decline if there is physical decay but this is not the only possibility. Anything which causes the *profitability* of capital equipment to decline will do just as well. Two possible causes of declining profitability have been identified in the literature:

1. If capital is used in fixed proportions with labour (a putty clay world), rising wages will cause older equipment to be discarded even if it is physically unchanged. As equipment ages, its profitability declines and it is discarded when profitability reaches zero. *Ex post* fixed proportions seem quite realistic for computers, where the rule is one worker, one PC. Suppose to the contrary that computer capital were malleable *ex post* and that each model is twice as powerful as its predecessor. Suppose too that the optimal capital/labour ratio is one worker to one PC of the latest type. Then the optimal ratio would be one worker to two older PCs, one worker to four PCs of the previous model, and so on. This is contrary to observation. Oulton (1995) shows that, in a putty clay world, the 'K' which should go into the production function is still one where machines are weighted by their relative marginal productivity or profitability, ie equation **(10)** still holds. Depreciation will not be geometric (since assets have a finite life) though geometric depreciation could still be a good approximation.
2. As capital ages, it may require higher and higher maintenance expenditure. This is particularly the case for computers and software, provided we understand maintenance in an extended sense to include maintenance of interoperability with newer machines and software. The profitability of a machine will then decline as it ages and it will be retired when profitability is zero. Whelan (2000a) has analysed the optimal retirement decision in such a world (although he assumes malleable capital).

Sometimes it is argued that only physical wear and tear should go into the measure of depreciation used to construct estimates of asset stocks. That part of depreciation which is due to obsolescence should be excluded. Computers (and software) do not suffer from wear and tear to any appreciable extent, so a large part of their high, measured depreciation rate must be due to

(14) Deviations from the mean rate of depreciation due to variation in the intensity of use have been estimated econometrically by Larsen *et al* (2002).

obsolescence. According to this argument, the growth rate of the US computer stock must be even faster than officially estimated by the BEA (Whelan (2000a)).

This argument is incorrect. Obsolescence, properly understood, is a valid form of depreciation. The reason is two-fold. First, as we have seen, asset mortality is part of the overall rate of depreciation. If an asset has been scrapped, then it cannot form part of the capital stock nor can it contribute to the VICS. Scrapping is of course an extreme result of obsolescence. Second, if the prices of surviving assets decline with age, then this means simply that the present value of the expected flow of services declines with age. At one level, it doesn't really matter what the cause is, the important point is that services are expected to decline. Decline could be for a whole host of reasons, including:

(a) wear and tear (which may be exogenous or may vary with use);
(b) rising costs of operating the asset; and
(c) a decrease in the *value* of the service flow even though the physical *quantity* of services is constant.

The last two possibilities are what is usually meant by obsolescence. Let us consider these in turn.

Suppose there are fixed coefficients: one person, one PC. Over time, wages are rising. Then the profitability of a PC of a given vintage will decline over time and eventually it will be scrapped when its quasi-rent falls to zero. Note that the physical capacity of the machine to produce output, and the value of that output, may be unchanging, but nevertheless the machine eventually gets scrapped because it ceases to be profitable. In this model, scrapping is endogenous: the faster the rate at which wages are rising, which depends on technical progress in the economy as a whole, the shorter the economic life of assets.(15)

The second type of obsolescence, declining value of services, could arise in the following way. Over time, new software is introduced which will not run on old machines. People prefer the new software, so the value of services from the old machines declines. Word 7 is better than Word 6 in most people's opinion, so a computer that cannot run Word 7 is worth less after Word 7 is introduced. There may be network effects here, but these do not affect the argument in the present context. You may be quite happy with Word 6 but are forced to change to Word 7 because everyone else has. But the value of the services of your old PC has still declined in the eyes of the market. And you have still voluntarily chosen to install Word 7 because you value your ability to communicate easily with other people.

Measuring depreciation in the presence of obsolescence and quality change(16)

Though this shows the correctness of incorporating obsolescence in the measure of depreciation, it is not so obvious how to do it in practice. The question is closely bound up with the issue of

(15) This is the vintage capital model of Solow and Salter. Oulton (1995) shows that a VICS can be calculated for this model in the same way as for a more neo-classical model. However, depreciation will not be exactly geometric since assets have a finite life here.

(16) This section draws on Oliner (1993) and (1994).

adjusting prices for quality change. Both issues can be addressed in principle by employing a properly specified hedonic equation. When quality is changing we must distinguish between transactions prices and quality-adjusted prices. The transactions price of a computer is the price of a box containing a certain model computer of some specified age. It cannot be directly compared with the transactions price of a different model since the quality of the two models may differ.

Suppose we had a panel of data on transactions prices of computers of different models, covering say two years. Then we could estimate the following regression:

$$\log \tilde{p}(i,s,t) = \beta_0 + \beta_1 z(i) + \beta_2 s(i,t) + \beta_3 YD + \varepsilon(i,t), \quad t = 1,2 \qquad \textbf{(46)}$$

where:

$\tilde{p}(i,s,t)$ is the transaction price (*not* quality adjusted) of the *i*th computer that is *s* years old in year *t* (the tilde is to indicate transactions rather than quality-adjusted price)
z(*i*) is some characteristic (say speed) which affects the perceived quality of computers. In practice, there would be a vector of characteristics.
YD is a year dummy (=1 in period 2).

Suppose that we have established that the regression is satisfactory from an econometric point of view. How should we interpret the coefficients? The coefficient on the year dummy, β_3, gives the rate of growth of computer prices in year 2, with quality (*z*) and age (*s*) held constant. So this coefficient gives the rate of growth of the *quality-adjusted* price of a new computer. This is just what we need to deflate investment in current prices to constant prices in order to estimate the stock of computers in units of constant quality.

The coefficient on age (*s*), β_2, which we expect to be negative, gives the factor by which price declines with age, holding quality constant. Ie, if δ is the geometric depreciation rate, then $1-\delta = \exp[\beta_2]$: however, this is to ignore asset mortality (see below). The specification is a bit restrictive, since it constrains the rate of depreciation to be the same at all ages, the geometric assumption again. It may be that life is more complicated, but this does not change the basic principles being illustrated here.

There is another adjustment we need to make to get true depreciation. The regression suffers from survivor bias. Some assets have been thrown away and so do not get to feature in the regression. Their price can be taken to be zero (assuming that scrap value and clean-up costs cancel out). If the proportion of assets of age *s* which survive at time *t* is $\phi(s,t)$, then a proportion $(1-\phi(s,t))$ has price zero. So the price of a model of age 1 at *t*, as a proportion of the price of a new version of the same model, is not $\exp[\beta_2]$ but $\exp[\beta_2]\phi(1,t)$ and this is the true depreciation factor. If the force of mortality is geometric, then this survivor-corrected rate of depreciation is also geometric.

Note that depreciation is defined at a point in time, just as before. It is the difference between (say) the price of a new Pentium 4 and the price a one year old Pentium 4, both prices being measured in (say) January 2003. The regression will also tell us what is the difference between the price of a new 386 and a one year old 386 at the same date, even if neither price is actually observed, because the 386 is no longer manufactured. The reason is that we can measure the characteristics of the 386 and use the regression to price it.

This equation says that depreciation is a function of a computer's age, but it may rather be a function of the age of the model of which it is a particular example. That is, a new Pentium 3 and a one year old Pentium 3 might sell for the same price at a point in time (in the absence of physical wear and tear), but both computers would fall in price, and by the same proportion, when the Pentium 4 is introduced. We could test for this by redefining a computer's age in the regression equation to be the number of years since that model was first introduced. Depreciation will now not be geometric but could still be approximately so. The reason is that examples of older models will on average have higher age than examples of younger models.

Note that depreciation and scrapping will be endogenous, just as in the Salter-Solow vintage capital model. The rate of depreciation will depend on technical progress in computers and software.

Estimating depreciation in practice

Empirically, estimates of both capital stocks and services are bedevilled by two major areas of uncertainty. First, we need to know the service lives of assets. Second, there is the choice of the appropriate pattern of depreciation, and the associated pattern of decay: should we use geometric, straight-line, hyperbolic or one-hoss shay depreciation?

Service lives

Little is known about the service life of assets in the United Kingdom. Till 1983, the official estimates of the capital stock were based on the work of Redfern (1955) and Dean (1964); see also Griffin (1976). Their estimates of services lives were in turn based on the life lengths used by the Inland Revenue for tax purposes, from a period before the tax system encouraged business firms to depreciate assets more rapidly in their accounts than would be justified by true economic lives (Inland Revenue (1953)). In 1983, the Central Statistical Office (the predecessor of the ONS) revised the service lives downwards, citing (unpublished) 'discussions with manufacturers' as its authority (Central Statistical Office (1985), page 201). Following a report commissioned from the National Institute of Economic and Social Research (Mayes and Young (1994)), this reduction was reversed. But at the same time two other changes were introduced. First, a new category of asset, 'numerically-controlled machinery', was introduced into the ONS's Perpetual Inventory Model (PIM) of the capital stock. This type of asset is assumed to have a very short life by comparison with other types of plant and machinery (about 5-7 years) and the proportion of investment devoted to this type is assumed to rise over time. Second, some plant and machinery is assumed to be scrapped prematurely; the rate of scrapping is assumed to be related to the corporate insolvency rate, which has been on a rising trend since the 1970s (West and Clifton-Fearnside (1999)). Both these changes have led to a progressive shortening of the average service life of plant and machinery (but not of buildings or vehicles) in the PIM since about 1979.

Clearly then the empirical evidence for service lives in the United Kingdom is weak.[17] This judgment is confirmed by the OECD. In its capital stock manual (OECD (2001b), Appendix 3) it lists four countries (not including the United Kingdom) for which service lives 'appear to be based on information that is generally more reliable than is usually available for other countries': the United States, Canada, the Czech Republic, and the Netherlands. It is noteworthy that in each of these countries service lives are lower than assumed in the United Kingdom for both buildings and plant and machinery (at least before the effects of premature scrapping are considered).

Evidence on the pattern of depreciation

No international consensus has yet been reached on the appropriate assumption to make about depreciation (OECD (2001b)). In practice, a variety of approaches has been used. In the United States, the Bureau of Labor Statistics (BLS) produces estimates of the 'productive capital stock' or VICS that assume a hyperbolic pattern of decay rates, arguing that these are more realistic than geometric decay. The Australian Bureau of Statistics follows a similar approach (Australian Bureau of Statistics (2001)). But this pattern is not based on any strong empirical evidence.[18] Statistics Canada on the other hand uses geometric decay. The BEA does not estimate the VICS but does produce wealth measures of the capital stock using geometric depreciation (Fraumeni (1997); Herman (2000)). Jorgenson and his various collaborators in numerous studies (eg Jorgenson *et al* (1987); Jorgenson and Stiroh (2000)) have assumed geometric depreciation and decay. By contrast the ONS in common with many other national statistical agencies employs straight-line depreciation in their net stock estimates. Their gross stock estimates in effect assume one-hoss shay. No official VICS measure is published though work is currently underway within the ONS to produce one on an experimental basis.

Geometric depreciation is well supported as a rule of thumb by studies of second hand asset prices (Hulten and Wykoff (1981a) and (1981b); Oliner (1993), (1994) and (1996)). These generally find that a geometric pattern of depreciation fits the data well, even though it is frequently possible to reject the geometric hypothesis statistically. On this basis geometric depreciation has been adopted as the 'default assumption' in the US national accounts (Fraumeni (1997)). By contrast, straight-line depreciation is inconsistent with the evidence on asset prices. The stylised fact about new and second hand-asset prices is that the age-price profile is convex (OECD (2001b)). But the straight-line assumption predicts that asset prices decline linearly with age. It also predicts that efficiency declines linearly with age, before falling abruptly to zero when the asset reaches the end of its service life,[19] a pattern that may well be thought unrealistic.

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(17) Knowledge will be improved when the results of the ONS's new capital stock survey are published (West and Clifton-Fearnside (1999)).

(18) If decay is hyperbolic, the services of an asset decline at an increasing proportional rate with age. The ratio of the services from an asset which is s years old to the services from a new asset is given by the formula $(n-s)/(n-\beta s)$ where n is the service life and β is a positive parameter. One reason often cited for preferring the hyperbolic to the geometric assumption is that under geometric the largest loss of efficiency occurs in the first period of an asset's life, which is often though unrealistic. By contrast, under hyperbolic decay the losses get proportionately larger as an asset ages. However this may be, the only evidence on declining efficiency comes from studies of asset prices. There is little or no basis for estimating the additional parameter which the hyperbolic assumption requires. In practice, the BLS chooses a value of this parameter which will yield an age price profile approximately equal to that implicit in the BEA's wealth estimates, the latter of course based on the geometric assumption.

(19) This is proved in Diewert and Lawrence (2000, equation 11.22, page 281).

On these grounds, we adopt geometric depreciation for the empirical work reported below. Nevertheless considerable uncertainty still attaches to the estimated rates for different assets. In principle, the hedonic approach described in the previous subsection can be used to estimate the rates, in conjunction with data on service lives. This approach can be thought of as a rather idealised account of the method actually used by the BEA.[20] But for many assets, there is inadequate data on second-hand prices or anyway no studies have been done. Where studies have been done, they are not always up to date: much of the evidence relates to the 1970s and 1980s (Fraumeni (1997); U.S. Department of Commerce (1999)). In addition there are some methodological problems, to which we now turn.

Overestimation of depreciation when quality change is important

Suppose we estimated the regression equation **(46)** of the previous subsection with the quality variable omitted. Then the coefficient on age would pick up not only the pure age effect but also the effects of changing quality. Older models have lower quality, so the coefficient on age would be biased downwards, ie it would be more negative. It would then be wrong to estimate the stock of eg computers using quality adjusted prices, while simultaneously using depreciation rates estimated from price data which are not quality adjusted.

This is the difference between what Oliner (1993) and (1994) calls ‘full’ and ‘partial’ depreciation; see also Cummins and Violante (2002). He argues that the BEA was guilty of this error in its estimates of computer stocks. However, there has been a major revision to BEA methods since he wrote. Oliner’s distinction between full and partial depreciation is referred to in a subsequent BEA methodology paper (U.S. Department of Commerce (1999)). And BEA estimates of stocks of computers and peripherals are based in part on his work.[21] The BEA’s depreciation rate for PCs is now based on a more recent study (Lane (1999)). But below we argue that this study in fact suggests a lower rate than the BEA’s.

What about other assets? The basis for the BEA’s estimates of depreciation rates for surviving assets are the two Hulten-Wykoff studies, one for structures and the other for equipment.[22] (Estimates of asset lives, which as we have seen are also influenced by obsolescence, come from other sources.) In the structures study, the main estimates did not include quality variables. But they did try adding two quality variables to their regression: primary structural material and ‘construction quality’ (which they argue is ‘presumably closely correlated to the availability and quality of ancillary equipment’). They also added population (derived from zip codes), which may affect land values (though land was excluded from the value of the buildings). None of these variables had much effect on the coefficient on age (their footnote 21). This suggests that quality change was not important for these assets over the period they studied.[23]

The published version of the Hulten-Wykoff equipment study does not give any details of the estimation method, so it is not clear whether their regressions included any controls for quality. But their discussion says that their estimates do not distinguish between pure ageing effects and ‘obsolescence’ (better referred to as quality change in our view). And Oliner refers to these

(20) See Fraumeni (1997) for a full account.

(21) Oliner’s work did not cover PCs, though the BEA used to apply his results for mainframes to PCs.

(22) See Hulten and Wykoff (1981a) and (1981b).

(23) Their data came from a survey of building owners carried out in 1972.

estimates as 'full' depreciation. So it is possible that the BEA estimates for non-computer equipment are more open to criticism on this ground and hence may overstate the depreciation rate. On the other hand, the assets actually studied by Hulten and Wykoff were machine tools, construction machinery, and autos.[24] Quality change is likely to be less important for these assets than for computers.

The case of computers

Although Oliner's work did not cover PCs, the BEA at one time applied his results for mainframes to PCs. More recently, the BEA has changed its method once again. For PCs they now rely on an unpublished study of fair market values of PCs belonging to a California-based 'large aerospace firm' (Lane 1999).[25]

For computers, two different methods were used by Lane to calculate fair market values. The first method was based on second-hand prices of computers from a variety of sources, including dealers. These were used to estimate the 'value factor' in the formula

Value = Original Cost x Value Factor

(See Lane (1999), page 12.) Note that 'Original cost' is the historic cost when the asset was new. The second method used the formula

Value = Replacement Cost New (RCN) less Normal Depreciation

where RCN is the price when new (original cost) adjusted for inflation. RCN is intended to be what an asset yielding 'comparable utility' (which we can interpret as comparable quality) would cost today. In practice this was estimated using the BLS price index for computers, which is of course adjusted for quality change. This leads to the formula:

Value Factor = RCN Factor x Percent Good

(See Lane (1999), page 17.) Here 'percent good' is the second-hand price as a proportion of the price new *at the same point in time* (not as a percent of original cost). In other words, it corresponds to the economist's concept of depreciation.

Under the first method, data were collected on the prices of a given model at various ages, which were then compared with its original cost. This information was obtained for a large number of models. Depreciation schedules showing the second-hand price as a percentage of the new price ('original cost') were plotted and a curve fitted to derive an average 'depreciation schedule' (using this term in the commercial sense, not the technical economics sense).

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[24] They also studied office equipment including (presumably) computers, but at least for computers their estimates have been superseded.

[25] In California, the property tax applies to equipment (including computers) as well as to real estate, and the base for the tax is fair market value. So there is considerable interest in valuing second-hand assets correctly. We are grateful to Richard Lane for sending us a copy of his report. The comments in the text should not be taken as critical of his study, whose focus was the correct market valuation of second-hand assets for tax purposes, not the estimation of economic depreciation in the national accounts.

The second method used a separate study of over 2000 PCs (no details given) to determine that the mean life of a PC was 34 months. The data on survival were then fitted to a theoretical survival curve (Winfrey S-0). Percent good was calculated as:

Percent Good = Annuity value of remaining service ÷ Annuity value of total service

Unfortunately, the public version of this report did not go into any detail as to how this calculation was actually done. But apparently the estimation took into account that utilisation declines over the asset's life (see Lane (1999), page 17). So percent good included some decline in service from surviving assets as well as the effect of the shorter expected life of ageing assets. The second method does seem to rest on more assumptions and on this ground the first is to be preferred.

The results are in Table E. The two methods produce similar but not identical results. Up to an age of three years, the percent good is very similar. For age above three years, percent good on the market data method is a good bit higher. At age five, percent good is 20% using market data and 6% using the survival curve approach.(26)

According to Herman (2000): 'The depreciation of PC's is now based on a California study of fair-market values of personal property including PC's [the Lane study]. The new estimates are based on a geometric pattern of depreciation that by the fifth year, results in a residual value for a PC of less than 10% of its original value. … The new method is consistent with the general procedure for calculating depreciation that was adopted in the 1996 comprehensive NIPA revision; assets are now depreciated using empirical evidence on used-asset prices and geometric patterns of price declines.' This suggests that the BEA is using the depreciation rates implied by the market data method in Lane's study.

Table E Value factors for low cost (<$50k) computers

2	*Based on market data*		*Based on survival curve*		
Age (years)	*Percent good (%)*	*Value factor (% of original cost)*	*RCN factor (%)*	*Percent good (%)*	*Value factor (% of original cost)*
	(1)	*(2)*	*(3)*	*(4)*	*(5)*
0	*100*	100	100	100	100
1	*70*	61	87	70	61
2	*47*	37	78	46	36
3	*36*	22	61	29	18
4	*30*	14	47	16	8
5	*20*	8	40	6	2
6	*15*	5	33	5	2
7	*10*	3	31	5	2

Source: Lane (1999), exhibits 5 (page 26) and 10 (page 32).
Note: Column (1), is not in Lane (1999), is calculated as 100 times column (2) divided by column (3).

(26) The results for more expensive computer systems were quite similar.

However, there is a problem. Herman's statement that the residual value of a computer is less than 10% after five years refers to column (2) ('Value factor') in the table above, not to column (1) ('Percent good'). If the geometric rate has been calculated from column (2), it would be about 40%. If from column (1), 'only' about 30%. In fact, one can calculate from the BEA's fixed asset Tables 2.1 and 2.4 that the depreciation rate on 'Computers and peripheral equipment' has been running at more than 40% since 1992 (Herman (2000)). But if (as in the present paper) we wish to define depreciation in the economic sense as the difference between the prices of a new and of a second-hand asset *at a point in time,* then we should be using the figures in column 1, not those in column 2. On this basis, the depreciation rate that the BEA is using for PCs is too high.

Conclusion on estimating depreciation in practice

1. Aggregate depreciation is the result of two forces: (a) the scrapping or retirement of assets and (b) the decline in the services yielded by a surviving asset as it ages. What is commonly called obsolescence may cause either premature scrapping or a decline in the value of services. Hence obsolescence is correctly counted as part of depreciation.
2. The hedonic pricing approach, which can be employed to measure quality adjusted prices, can also be used to measure depreciation in the presence of obsolescence. Such studies underlie the BEA's estimates of depreciation rates, including for computers.
3. While the BEA methodology seems sound in principle, the empirical evidence on depreciation is still somewhat patchy and in some cases out of date. Also it is possible that depreciation rates for non-computer equipment (insofar as they are intended to measure economic depreciation as defined in the present paper) have been overstated by the BEA, since the studies on which the BEA relies may not have controlled completely for quality change.
4. Computers (though not software) have been the subject of numerous studies which do control for quality change. The BEA has recently changed the depreciation rates used for PCs as a result of a new study. But their new depreciation rate is too high as a measure of economic depreciation: the same study from which their estimate of about 40% is derived supports a figure of about 30% as the rate of economic depreciation.

4 Capital stocks, VICS and depreciation: sources, methods and results

Sources and methods for quarterly and annual estimates of the wealth stock and VICS

The data

This section describes the sources and methods used to construct quarterly (seasonally adjusted) and annual estimates of asset stocks, the wealth measure of the aggregate capital stock, and the VICS, for the whole economy. Further details are in Appendix B.

The wealth and VICS measures are conceptually and data consistent. Both measures assume geometric depreciation and are based on the same underlying ONS series for gross investment. The only difference between them is that in weighting together the growth rates of the asset stocks, the wealth measure uses shares in the aggregate value of assets (asset price weights), while the VICS uses shares in aggregate profits (rental price weights).

Following the ONS breakdown, five types of asset are distinguished initially:

1. Other buildings and structures ('Buildings')
2. Transport equipment ('Vehicles')
3. Other machinery and equipment and cultivated assets ('Plant and machinery')[a]
4. Intangible fixed assets ('Intangibles')[b]
5. Inventories ('Stocks')

a. Includes computers and some of software investment.
b. Includes some of software investment.

Note that dwellings are excluded. The VICS and wealth measures that we present below exclude inventories. But the VICS measures are still influenced by inventories since the latter affect the estimates of the rental price weights. In effect we assume that inventories earn the same nominal rate of return after tax as do the other assets.

The official investment series include investment in computers and software but these are not distinguished separately. For some of our measures we thought it important to see the effect of greater disaggregation by asset type. We therefore developed our own estimates of computer and software investment, building on earlier work (Oulton (2001a)). These estimates derive mainly from the annual supply and use tables, supplemented by earlier input-output tables. The methods and sources are discussed below; software investment is discussed more fully in Appendix C and computer investment in Appendix D. An important issue is: given a series for real investment in computers, and a series for total real investment in plant and machinery, ie including computers, how should one derive an index for real investment in plant and machinery *excluding* computers? This is discussed in Appendix D; note that simply subtracting computer investment from the total is the wrong answer.

Method

Quarterly estimates

When rental prices were calculated using the quarterly model, the estimates of the rental price weights were excessively volatile. So we adopted a two stage procedure: (1) estimate the rental price weights using *annual* data; (2) estimate the quarterly VICS using *quarterly* capital stock data and *annual* rental price weights. Even on annual data, the rental prices needed some smoothing (see below).

To estimate the quarterly VICS, the *quarterly* growth rates of asset stocks are weighted together using the *annual* rental price weights. But here we revert to the simpler model of Oulton (2001b) and now assume that capital services in period t are proportional to asset stocks in place at the end of t-1:

$$K_{it} = A_{i,t-1}, \qquad i = 1,...,m \tag{47}$$

We also measure the growth rate of the wealth measure on an end-of-period basis:

$$\ln\left[A_t / A_{t-1}\right] = \sum_{i=1}^{m} \bar{v}_{it} \ln\left[A_{it} / A_{i,t-1}\right],$$
$$\bar{v}_{it} = (v_{it} + v_{i,t-1})/2, \;\; v_{it} = \frac{p_{it}^{A} A_{it}}{\sum_{i=1}^{m} p_{it}^{A} A_{it}}, \;\; i = 1,\ldots,m \qquad \textbf{(48)}$$

Consequently, when comparing the quarterly (but not the annual) VICS and wealth measures, *the latter should be lagged one quarter*.

Cyclical scrapping
So far we have assumed that the depreciation rate for each asset type does not vary over time. Each rate includes an allowance for scrapping at some 'normal' rate. But arguably assets are more likely to be scrapped in a recession and certainly this is consistent with much anecdotal evidence.(27)

One might argue that the life of capital assets is prolonged during a boom: assets are not scrapped when they normally would be but are retained in order to meet high demand. However obsolescence may be more rapid during a boom, ie lives are shorter, since firms are happy to buy the latest kit; this may be particularly the case for high-tech assets. On the other hand during a recession firms who wish to maintain their capital at its current level may be more cautious about replacing it at the normal rate, due to financial constraints and higher perceived risk, so lives get longer. We have no evidence on the relative strength of these opposing tendencies.

We assume that the mechanism is asymmetric. Plant and machinery (including computers and software) is assumed to be scrapped at an accelerated rate when output falls, but there is no corresponding mechanism when output rises. The effect is only temporary: the assets which get scrapped would have been scrapped in the end anyway. So this mechanism makes the time path of the plant and machinery stock more cyclical. Buildings and vehicles are assumed not to be subject to cyclical scrapping. Buildings may stand idle but in a recession they are not assumed to be physically destroyed more rapidly than normal. One justification is that buildings, like vehicles, usually have broad second-hand markets, hence scrapping would not generally be profitable. Plant and machinery on the contrary is frequently highly specialised and second-hand markets are thin. Also, if a machine can be sold, it may be for export abroad, in which case it ceases to be part of the UK capital stock.

If the fall in output is expected to be only temporary, then it would be irrational to scrap capital. But if the fall is expected to be *permanent*, it is reasonable that firms would adjust their capital stocks proportionately. We interpret a fall as permanent when it is a fall when measured on an annual basis. From 1973 to 2001, there were nine years in which manufacturing output [ONS code CKYY] fell. The largest fall was in 1980, 9.1%. But one can calculate that non-manufacturing output has never fallen on an annual basis over this period. Hence the cyclical scrapping mechanism is assumed to apply only to manufacturing. Let p be the proportion of the plant and machinery stock which is located in manufacturing: between 1973 and 1999, this proportion fell from 43% to 30%. Then in years when output falls in manufacturing, our model

(27) This issue attracted attention following the recession of 1979-81 which was particularly deep in manufacturing. The widely varying estimates of premature scrapping in that period are reviewed in Oulton and O'Mahony (1994, chapter 3).

assumes that the whole-economy plant and machinery stock (including computers and software) is reduced by p times the fall in manufacturing output.

The sensitivity of our estimates to this cyclical scrapping assumption is explored further below.

Estimates of capital stocks and VICS

In this subsection we present our estimates of wealth and VICS constructed under a range of assumptions about depreciation and asset lives. The estimates of growth rates are calculated using equations **(36)** and **(37)** in Section 2. The estimates of levels in constant prices are calculated by assuming that the nominal and real values are the same in 1995 Q2 and then applying the growth rates to these values. 'Whole-economy' real growth rates of wealth and VICS are chain-weighted aggregates of asset level growth rates, ie they are Törnqvist indices. The weights are the nominal wealth shares in the wealth measure and the nominal profit shares in the VICS measure. All our measures assume that depreciation is geometric.

The variants

We have constructed six variants of each of our wealth and VICS measures. Since, as we have seen, there is considerable uncertainty about the true length of asset lives, our assumptions are designed to illustrate the extent of the corresponding uncertainty about the level and growth rate of capital. A second aim is to show the effect of a more detailed disaggregation by asset type. The variants are described in the table overleaf.

These variants share a common dataset — the underlying investment and profits series are identical inputs into both. Unless stated otherwise, all data (including investment price deflators) are consistent with the UK national accounts. The dataset is described more fully in Appendix B. The UK national accounts provide quarterly constant and current price investment data at a five-asset level: dwellings, buildings, other machinery and equipment (ie, plant and machinery), transport equipment (ie, vehicles) and intangible fixed assets. Dwellings are excluded from all our calculations.

The choice between wealth and VICS measures depends on the purpose at hand. The six variants for each measure are designed to throw light on the quantitative importance of methodological and data uncertainties. To see the effect of different asset life assumptions, variants BEA, ONS1 and ONS2 were constructed. Variant BEA is constructed using the four principle assets and uses the BEA's asset lives and corresponding (geometric) depreciation rates. Variant ONS1 is also constructed using the four principle assets but employs the official (ONS) asset lives combined with US estimates of the declining balance rate in the formula for the (geometric) depreciation rate.[28]

[28] The formula for the depreciation rate is R/L where R is the 'declining balance rate' and L is the asset life (see page 27). See Table B.2, Appendix B for the depreciation rate calculations for ONS1.

	Variant	Description[29]
1	BEA	Aggregated from four assets: buildings, plant, vehicles, intangibles Depreciation Rates: consistent with US NIPA ('BEA consistent')
2	ONS1	Aggregated from four assets: buildings, plant, vehicles, intangibles Depreciation Rates: calculated using asset lives consistent with UK national accounts ('ONS consistent')
3	ONS2	Aggregated from four assets: buildings, plant, vehicles, intangibles Depreciation Rates: geometric rates equivalent to ONS straight-line rates
4	ICT1	Aggregated from five assets: buildings, plant, vehicles, intangibles, computers Depreciation Rates: consistent with US NIPA ('BEA consistent') Computer price index: consistent with UK national accounts
5	ICT2	Aggregated from five assets: buildings, plant, vehicles, intangibles, computers Depreciation Rates: consistent with US NIPA ('BEA consistent') Computer price index: US computer price index, adjusted for exchange rate changes
6	ICT3	Aggregated from six assets: buildings, plant, vehicles, intangibles, computers, software Depreciation Rates: consistent with US NIPA ('BEA consistent') Computer price index: US computer price index, adjusted for exchange rate changes Software price index: US price index for pre-packaged software, adjusted for exchange rate changes

The wealth measure of ONS1 comes closest in spirit to the official, net stock measure of the UK capital stock. However, there are three differences. First, in the ONS model, the service life of particular asset can vary across industries, whereas we use the same life in all industries. Second, to aggregate capital stock across asset types, the ONS simply sums the stocks. We on the other hand use chain-linking (as will the ONS from 2003). Thirdly, the ONS assumes straight-line depreciation whereas we assume geometric.

Variant ONS2 is identical to variant ONS1 except that the depreciation rates for buildings and plant are calculated as 'steady-state values'. Section 3 (Table B) provided a motivation for the use of these steady-state values.

ONS series for investment do not break out investment in computers or software separately. Computers are subsumed in the plant category and software is split between plant and intangibles. To see if asset composition has an important effect on the aggregate measures, variants ICT1, ICT2 and ICT3 were constructed.

(29) In variants ICT1-ICT3 plant is actually 'rest of plant' and in variant ICT3, intangibles is actually 'rest of intangibles'. The reason for this is explained on the next page.

Variant ICT1 treats computers as a separate asset, uses BEA consistent asset lives and uses the UK investment price deflator for computers.(30)(31) The plant category is now just the 'rest of plant' to avoid double counting.

Variant ICT2 is identical to variant ICT1 in all aspects except that it uses the official US (BEA) price index for computers, adjusted for changes in the sterling dollar exchange rate. As mentioned in Section 3, this index is constructed using hedonic techniques.

Variant ICT3 builds on variant ICT2 by breaking out software as a separate asset (in addition to computers). We use a BEA research series for the software investment deflator, the price index for pre-packaged software, (32) and apply the 'times 3 adjustment' as described in Oulton (2001a). The plant category is now the 'rest of plant', ie computers and software are excluded, and the intangibles category is now the 'rest of intangibles' after excluding the part of software previously included here. Obtaining software series for investment and then apportioning it to plant and intangibles investment is described in Appendix C. The methodology adopted to calculate the 'rest of plant' and the 'rest of intangibles' in constant prices is described in Appendix D.(33)

Table F: Depreciation rates for each variant by asset (per cent per annum)

Variant	Buildings	Plant and Machinery	Vehicles	Intangibles	Computers	Software	*Price index for computers /software*
BEA	2.50	13.0	25.00	22.0	--	--	--
ONS1	1.14	5.69	20.59	22.0	--	--	--
ONS2	2.03	7.57	20.59	22.0	--	--	--
ICT1	2.50	13.0	25.00	22.0	31.50	--	UK
ICT2	2.50	13.0	25.00	22.0	31.50	--	US
ICT3	2.50	13.0	25.00	13.0	31.50	31.50	US

The annual geometric depreciation rates used in the variants are shown in Table F. (34) The depreciation rate on intangibles is kept at the BEA rate of 22% per annum for all the variants except variant ICT3 which treats software as a separate asset. In the UK national accounts, part

(30) ONS code: PQEK.
(31) With Blue Book 2003, the aggregate ONS capital stock series will consider computers separately as an asset (giving them a shorter life length of five years) but the investment deflator for computers will be the same as that for plant and machinery as a whole.
(32) Parker and Grimm (2000).
(33) In nominal terms, separating out computers and software from plant and software from intangibles to get 'rest of plant' and 'rest of intangibles' is easy: simply subtract the investment in these subcategories from the total. However, to get constant price series additional calculations are needed because for their constant price estimates the ONS changes the weights about every five years.
(34) The analysis always refers to the annual depreciation rates. However, these annual rates are converted to quarterly rates for the calculations since the data are on a quarterly frequency.

of software is in intangibles and part in plant so separating software from the intangibles basket will correspondingly lower the depreciation rate on the rest of intangibles.[35]

The results: an overview
Estimates of levels and growth rates are presented for the period 1979 Q1-2002 Q2, though the calculations were actually done from an earlier date.[36] Average growth rates of the aggregate wealth and VICS measures are given in Table G and standard deviations of these growth rates are given in Table H. Average shares[37] in wealth and profits are shown in Table I and the growth rates of the individual assets are given in Table J. Graphs of the shares over the entire sample period are presented in Appendix E.

Considering first the wealth measure, we find little difference across variants in the average growth rate over the whole period 1979 Q1-2002 Q2 (Table G). The growth rates do not appear much affected by differences in asset composition or asset lives. This is because buildings have by far the largest share in the wealth measure but show little difference in their average growth rate across variants. Whatever variation there is appears to be caused by changes in the growth of plant stock (Graphs E.1 and E.2 in Appendix E plot the growth rates of buildings and plant for variants BEA and ICT3 as an example).

The VICS measure on the other hand shows larger differences between variants. This is because the VICS measures give greater weight to assets whose rental prices are high in relation to their asset prices and which are growing rapidly. Thus it is not surprising that VICS variant ICT3 has the highest average growth rate. Over 1990 Q1-2002 Q2, its quarterly growth rate was some 0.4 percentage points faster than that of wealth variant ICT3. Variant ICT3 treats computers and software separately. These assets have short service lives and falling asset prices, consequently their rental price is high relative to their asset price. This combined with rapid growth in these stocks (Table J) means that the VICS, which weights by rental price, grows more quickly than the wealth measure. This story is particularly true of the 1990s, where computers and other high-tech assets have become increasingly important.

VICS variant BEA, while having a slightly higher average growth rate in the 1990s than the corresponding wealth measure, does not show the complete picture. This is because it does not recognise that the asset composition mix in the 1990s had shifted to fast growing assets with shorter lives. The combined rental weight of ICT in variant ICT3 over the entire period is 8% on average (Table I). This multiplied by high growth in these assets (Table J) gives VICS variant ICT3 an added boost.

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(35) Variants ICT1-ICT3 treat computers as a separate asset. As mentioned earlier, computers are included in plant and machinery investment by the ONS, so separating them out from plant should arguably lower the depreciation rate for the remainder of plant. However, the depreciation rate used in the United States by the BEA for plant and machinery excluding computers and software is about 13% for the entire time period (including the 1990s), so we use this rate for the rest of plant in Variants ICT1-ICT3.
(36) The calculations are actually done for from 1965 Q1 onwards (with starting values for the asset level stocks set in 1962 Q1) for variants BEA, ONS1 and ONS2. Similarly, calculations for ICT1-ICT3 are done from 1976 Q1 onwards. Estimates are presented from 1979 Q1 onwards because by this time the impact of the initial values of the stocks (which derive from historic data of lower quality) is minor, or even negligible in the case of the short-lived assets. The wealth and VICS estimates under a variety of assumptions can be downloaded from the Bank of England's website (www.bankofengland.co.uk/workingpapers/capdata.xls).
(37) Shares may not add up to 100 due to rounding.

The most striking contrast in Table G is between wealth measures which implicitly use UK price indices to deflate ICT assets (BEA, ONS1, ONS2, and ICT1) and VICS measures which employ US methods (ICT2 and ICT3). Thus we find that in the 1990s the quarterly growth rate of the ICT3 VICS measure was 0.3 percentage points higher than that of the BEA wealth variant.

Table G: Average growth rates (per cent per quarter, chain-linked)

Variant	*1979 Q1-2002 Q2*		*1979 Q1-1989 Q4*		*1990 Q1-2002 Q2*	
	Wealth	**VICS**	**Wealth**	**VICS**	**Wealth**	**VICS**
BEA	0.76	0.81	0.71	0.69	0.80	0.91
ONS1	0.79	0.81	0.74	0.69	0.83	0.91
ONS2	0.73	0.76	0.65	0.62	0.79	0.88
ICT1	0.67	0.78	0.68	0.71	0.66	0.84
ICT2	0.68	0.87	0.70	0.82	0.67	0.92
ICT3	0.72	1.04	0.74	0.98	0.71	1.10

Table H: Standard deviation of growth rates (per cent per quarter)

Variant	*1979 Q1-2002 Q2*		*1979 Q1-1989 Q4*		*1990 Q1-2002 Q2*	
	Wealth	**VICS**	**Wealth**	**VICS**	**Wealth**	**VICS**
BEA	0.22	0.38	0.26	0.43	0.15	0.30
ONS1	0.17	0.31	0.21	0.34	0.12	0.24
ONS2	0.20	0.34	0.24	0.37	0.13	0.26
ICT1	0.21	0.34	0.25	0.40	0.17	0.25
ICT2	0.21	0.38	0.25	0.43	0.18	0.32
ICT3	0.23	0.45	0.26	0.50	0.19	0.39

Table I: Average shares in nominal wealth (W) and profits (P) by asset: 1979 Q1-2002 Q2

(per cent)

Variant	**Buildings**		**Plant**		**Vehicles**		**Intangibles**		**Computers**		**Software**	
	W	**P**	**W**	**P**	**W**	**P**	**W**	**P**	**W**	**P**	**W**	**P**
BEA	71	47	24	39	4	11	1	3	--		--	
ONS1	65	44	30	43	4	11	1	3	--		--	
ICT1	70	48	23	34	4	11	1	3	1	4	--	
ICT2	70	48	23	34	4	11	1	3	1	4	--	
ICT3	70	46	23	33	4	11	1	2	1	4	1	4

Table J: Average growth rates of asset stocks: 1979 Q1-2002 Q2 (per cent per quarter)

Variant	**Buildings**	**Plant**	**Vehicles**	**Intangibles**	**Computers**	**Software**
BEA	0.70	1.04	0.18	0.94	--	--
ONS1	0.73	0.98	0.19	0.94	--	--
ICT1	0.70	0.47	0.18	0.94	5.21	--
ICT2	0.70	0.47	0.18	0.94	7.25	--
ICT3	0.70	0.45	0.18	0.91	7.25	5.89

The VICS measures have higher average growth rates than their wealth counterparts and their growth is also more volatile, when volatility is measured by the standard deviation (Table H). This higher volatility does not have a simple explanation. Partly it is due to the more volatile assets receiving larger weight in the VICS, partly to the fact that profit shares are more dispersed than wealth shares,(38) and partly to an interaction between shares and growth rates.

Across variants, shares are also influenced by differences in assumed depreciation rates, since the latter cause changes in the level of the asset stocks which influence the calculation of the shares. Thus in Table I when the depreciation rate on plant and buildings changes between the BEA and ONS1 variants, the wealth and profit shares also change. Separating out computers changes the profit share of plant more than its wealth share (BEA versus ICT1). Two forces are at work here. First, a change in the depreciation rate for 'rest of plant' causes some change in the 'rest of plant' stock which affects both measures. But second, the depreciation rate has an additional impact through the formula for the rental price, equation **(34).**

As we will see in more detail below, constant price levels of wealth do not differ substantially across variants when the asset composition or the deflators change (Chart 9). However, in the discussion of straight-line versus geometric depreciation in Section 3 it was pointed out that changes in the asset level depreciation rates will affect the levels of the aggregate stock. This shows up clearly when we compare the levels for variants BEA, ONS1 and ONS2 (Chart 5).

In the remainder of this subsection we look at the sensitivity of the estimates in more detail. Specifically, we consider their sensitivity to (a) variations in asset life; (b) separating out computers and software; (c) the choice of price index for computers; (d) the method of aggregation, chain-linking or fixed-base; and (e) cyclical scrapping.

Sensitivity of estimates to asset life assumptions

In Section 3 we discussed straight-line as an alternative to geometric depreciation. In the US NIPA, depreciation is assumed to be (in most cases) geometric (Fraumeni (1997)), while the ONS (along with many other national statistical agencies) assumes straight-line depreciation. A related issue is the service life assumed for each asset. In general, the service lives assumed by the ONS are longer than in many other countries, as we have seen.

Variant BEA uses US geometric depreciation rates whereas ONS1, while geometric, uses UK asset lives. (See Table B.2 in Appendix B for the calculations.) Charts 1 and 2 show that using longer, UK asset lives does not have a significant impact on the growth rate of wealth and VICS.

As pointed out in Section 3, quantitative comparisons between straight-line and geometric depreciation are not easy because in the straight-line case, the depreciation rate depends on the age structure of the stock. But if we assume that investment in an asset is growing at a constant rate over the assumed life of the asset, then we can calculate a steady-state depreciation rate in the straight-line case that can be used to compare to the geometric case.(39) The longer UK asset lives

(38) See Charts E.5 -E.16 in Appendix E for a graphical representation of this.

(39) Clearly the comparison is trivial if the steady-state depreciation rate associated with the straight line case is the same as the geometric rate.

coupled with the straight-line assumption means that the steady state depreciation rates corresponding to these asset lives are smaller than the BEA consistent geometric rates. (See Table B, Section 3.)

Chart 1

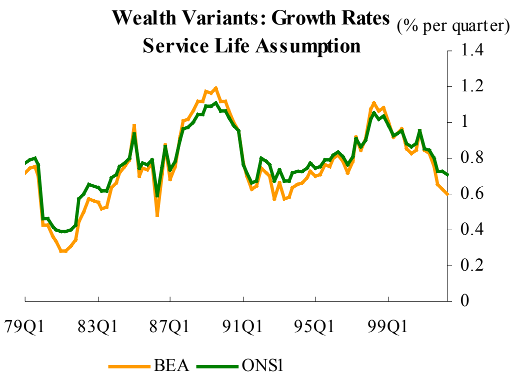


Chart 2

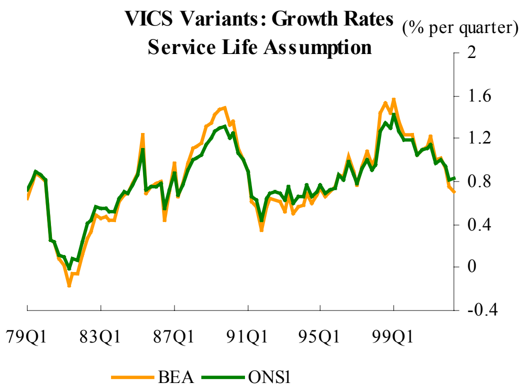


Chart 3

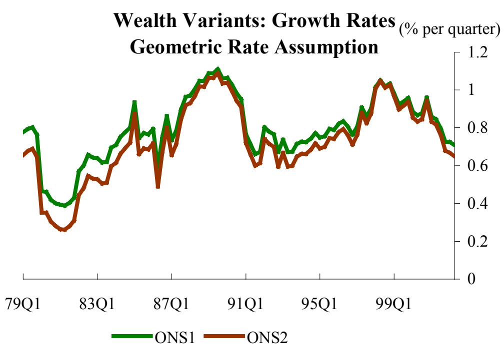
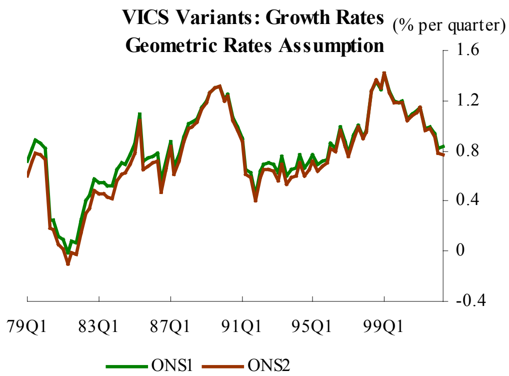


Chart 4



Variant ONS2 uses steady-state values for buildings and plant instead of the declining balance values used in ONS1.[40] Charts 3 and 4 compare wealth and VICS growth rates for these variants. They show that even large differences in asset life assumptions (eg, for plant) do not appear to make much of a difference to the aggregate growth rates, at least towards the end of the sample period.

However, different depreciation rates have a substantial effect on the *levels* of the stocks, as was noted in the discussion of straight-line versus geometric depreciation in Section 3. If the rates used to estimate UK asset stocks were the higher US ones, the level of the stocks would be lowered by a large amount. This is evident from Chart 5 when comparing BEA to ONS1/ONS2: higher depreciation rates shift the whole profile of wealth downwards.

[40] Since vehicles and intangibles already had relatively high depreciation rates in ONS1, they were kept unchanged in ONS2.

Chart 5

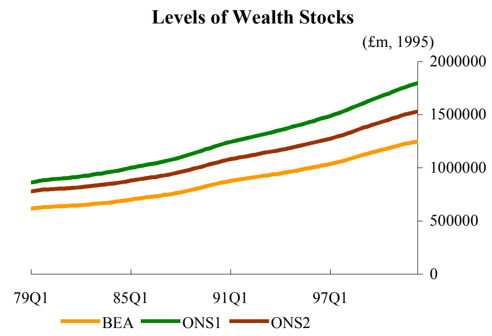
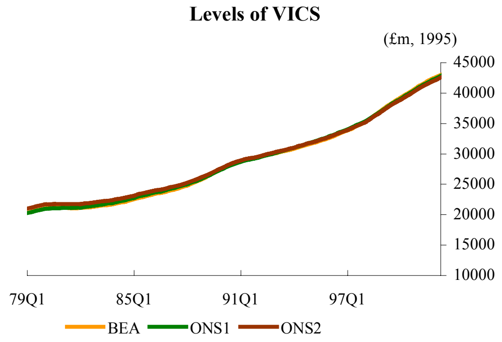


Chart 6



The effect on the level of the VICS is necessarily smaller (Chart 6). This is because in all variants the real level of the VICS in the base period (1995 Q2) is equal to the nominal level (profit in current prices) in that period. So different assumptions about depreciation simply rotate the VICS profile about the fixed point in 1995. In other words, before and after the base year the estimated level will be affected by different assumptions about depreciation only to the extent that the growth rates are affected. By contrast, though for all variants nominal and real investment levels are equal in 1995 Q2, there is no comparable constraint on the real and nominal level of wealth: these can and do differ in the base period.

Asset composition: separating out computers and software

We start by comparing BEA with ICT1 in Charts 7 (wealth) and 8 (VICS). The depreciation rates on all 'traditional' assets are the same, but in ICT1 computers have been separated out from the plant and machinery category and now are depreciating at a higher rate.[41] Separating out computers from plant has the apparently odd effect of lowering the growth rate of the aggregate wealth measure. To understand this, recall that computers are subsumed in the plant category for variant BEA (and for ONS1 and ONS2) and software is subsumed in intangibles and plant in all variants except ICT3. The average quarterly growth rate of plant (including computers) is around 1% (BEA, ONS1 and ONS2). When we separate computers (and/or software), the average quarterly growth of 'rest of plant' falls to about 0.5% (Table J). In fact, since 2001, the growth rate of 'rest of plant' is negative. The impact of the rapid growth of computers is counterbalanced by the small share of computers in wealth (1%), so the overall effect is to drag down the growth rate of the wealth measure when computers and/or software is included. The impact of this drag stemming from the 'rest of plant' is less severe in the VICS measures because of the growing share of computers and software in profit; the latter interacts with the very rapid growth of ICT asset stocks.

On the VICS measure, variant ICT1 (as compared to variant BEA) is also affected by the drop in the growth rate of the 'rest of plant' asset. But this is offset to a large extent by computers which are not only growing fast but also have a growing share in nominal profits.

[41] In Variant BEA they had a depreciation rate equal to that of plant (13%). In Variant ICT1, the rate is 31.5%.

Chart 7

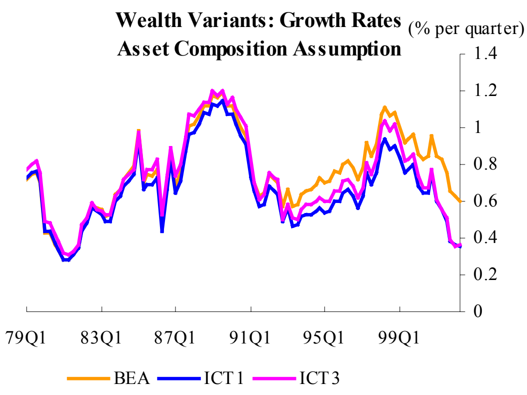
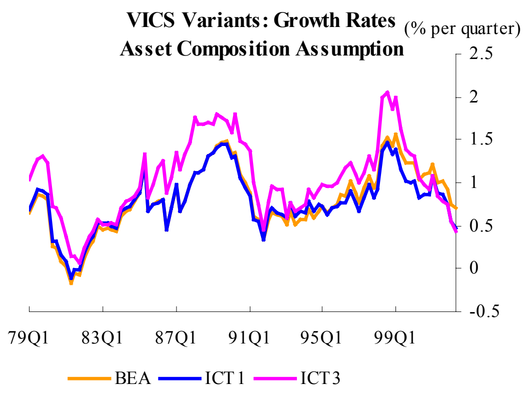


Chart 8



Software is another high depreciating, fast growing asset with a high rental to asset price ratio. While a strict comparison between ICT1 and ICT3 cannot be made,[42] it is interesting to see the impact of treating software separately. Growth rates of both wealth and VICS are higher though VICS shows the larger increase, especially in the mid to late 1990s. In level (real wealth) terms, there is not much of a difference (Chart 9). The higher BEA growth rates in the 1990s let the BEA estimate of the wealth level catch up with the ICT3 estimate. A similar picture emerges for the corresponding VICS levels (Chart 10).

Chart 9

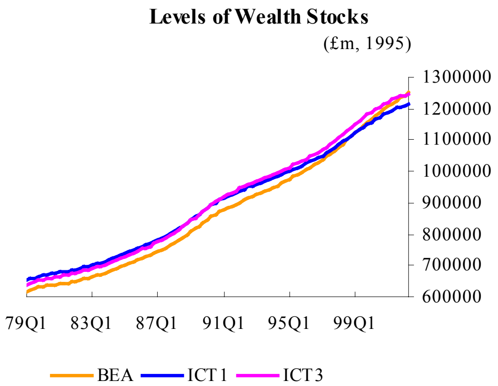
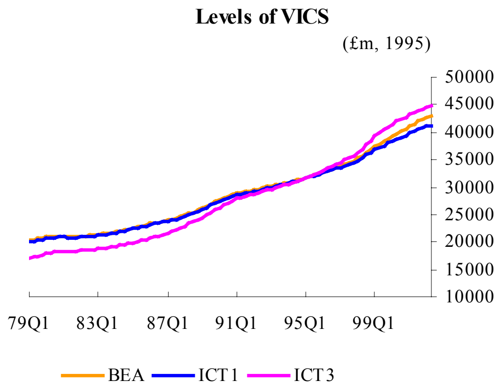


Chart 10



Now compare variants ICT2 and ICT3. They use the same (US, hedonic) price deflator for computers but variant ICT3 separates out software as an asset. As expected, the growth rates of the wealth measures show little difference (Chart 11) because even though software is growing fast, it has a very small wealth share. On the VICS measure, variant ICT3 shows strong growth in the mid to late 1990s (Chart 12), as result of rapid growth in the software stock and because the share of software in profits is higher than in wealth.

[42] ICT3 uses a different investment price deflator for computers.

Chart 11

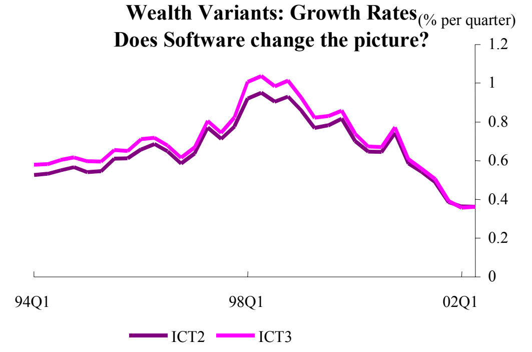
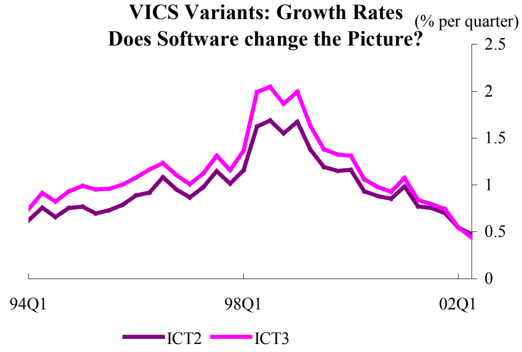


Chart 12



Sensitivity of the estimates to the price index for computers

The sensitivity of the wealth and VICS estimates to changes in the investment price deflator for computers follows a familiar pattern. Variant ICT1 uses the official UK national accounts producer price index for computers whereas variants ICT2 (and ICT3) use the US (hedonic) price deflator. The US computer investment price deflator has fallen faster than the UK one – hence when growth in the volume measure is considered, the asset stock is growing faster in variant ICT2 than ICT1 (see Table J). This larger growth, *ceteris paribus*, increases the growth of the aggregate measure. The impact is however muted in the wealth measure because the share of computers in wealth is very small (Chart 13). But as Chart 14 shows, the effect on VICS variant ICT2's growth rate is larger because of the share of computers in profit is greater than in wealth. The difference in wealth levels is negligible.

Chart 13

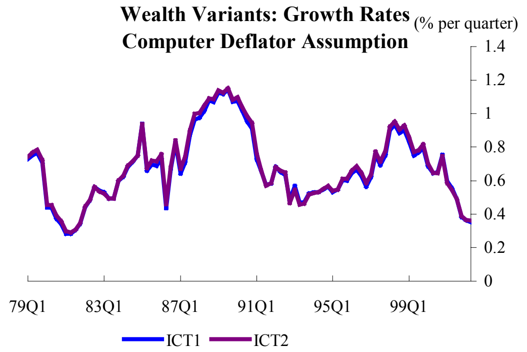
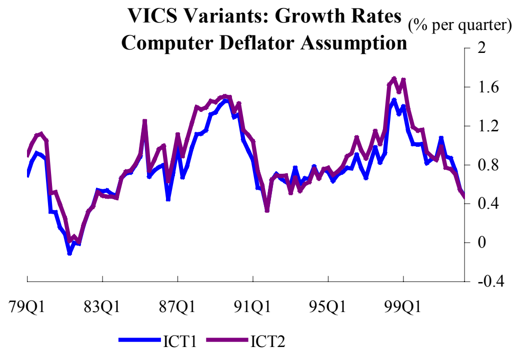


Chart 14



Sensitivity of the estimates to the method of aggregation: chained or fixed weights

The estimates presented so far are all quarterly chain-linked. It is generally agreed that fixed weight indices can be highly misleading when relative prices are changing rapidly. Current ONS methodology[43] is to change the weights every five years or so for most variables.[44] At the moment, the national accounts use 1995 weights for measuring growth rates from 1994 onwards.

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(43) See ONS (1998), page 221.

(44) The ONS capital stock is a simple sum of asset level stocks. With the ONS shifting to annual chain-linking in 2003, the official capital stock series will also be chain-linked then.

Over this period the relative price of computers has been falling rapidly, even employing the UK price index. To see the effect of chain-linking, we calculated ‘fixed but periodically updated’ indices of wealth and VICS (following ONS methodology), for all variants. Comparisons of the ONS1, BEA and ICT3 variants are presented in Charts 15-20.

For variants ONS1 and BEA, chained and fixed-weight aggregates are similar (Charts 15a, 15b, 16a, 16b). This is because the assets that experienced large relative price changes (computers, software, telecommunications) are subsumed in the larger asset categories and relative price changes between buildings on the one hand and plant and machinery or vehicles on the other have not been large.

Chart 15a

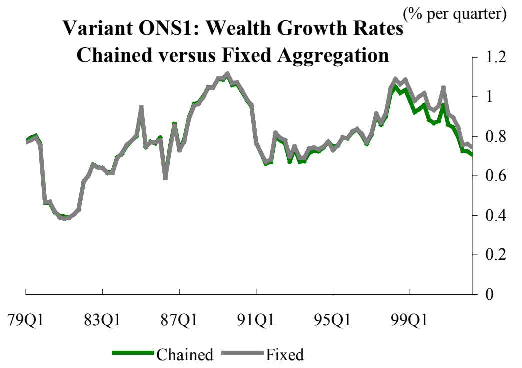


Chart 15b

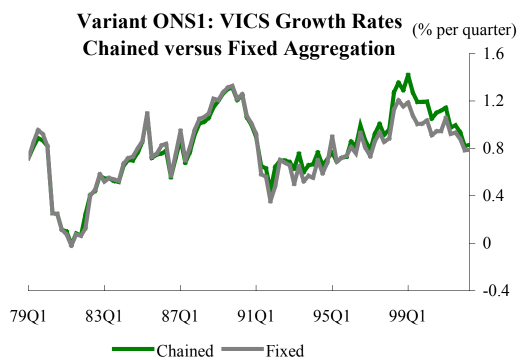


Chart 16a

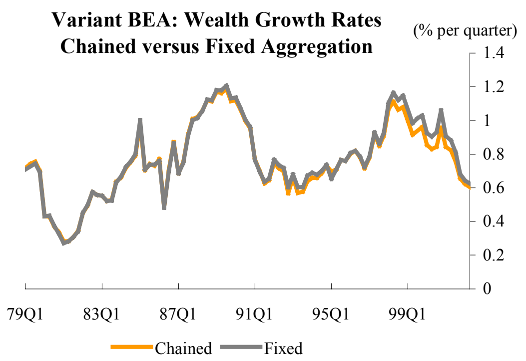
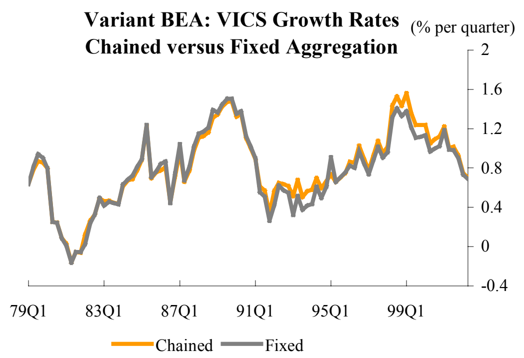


Chart 16b



The same cannot be said for variant ICT3 (Charts 17,18). Both the wealth and VICS growth measures show that the chained growth rates are lower for the most part than the fixed-base ones, as we would expect. The latest fixed base in use by the ONS is 1995. Since 1995, the investment price of computers and software has fallen rapidly. The fixed-base aggregate uses a share calculated on the higher 1995 prices (relative to the period that followed).

Chart 17

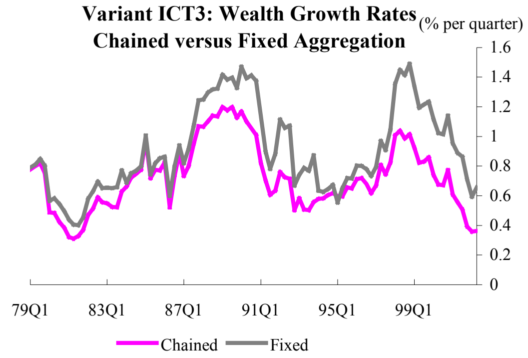
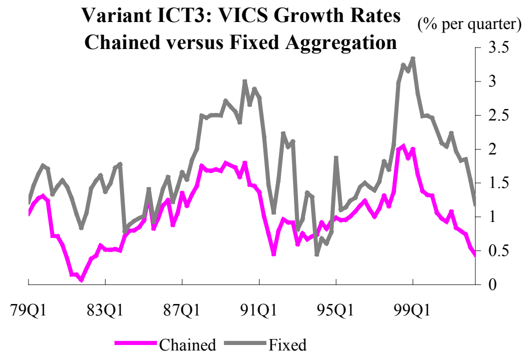


Chart 18



A similar consistent picture emerges in the levels context. Reflecting the high level of aggregation, chained and fixed-base levels of the real wealth measures of variants ONS1 and BEA show little difference (Chart 19a, 19b). Large price movements in certain assets create a divergence in levels of chained and fixed-base measures of variant ICT3 (Chart 20).

Chart 19a

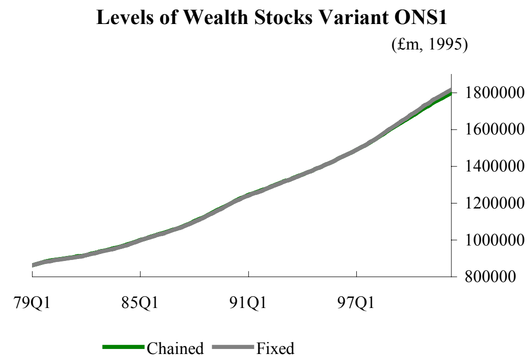


Chart 19b

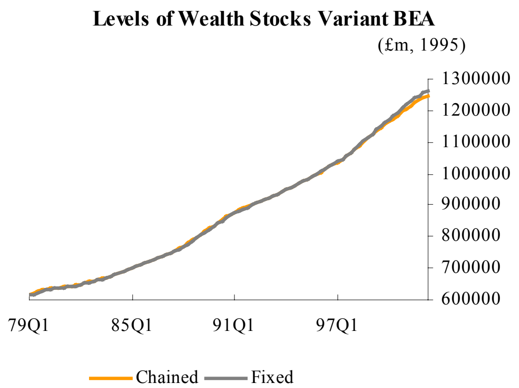
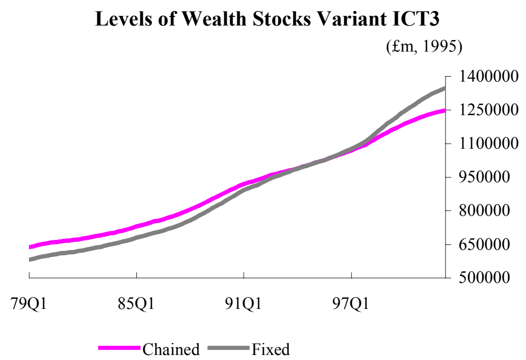


Chart 20



The cyclical scrapping assumption

All variants have embedded in them a cyclical scrapping mechanism, described more fully earlier in this section. This affects only plant and machinery, computers and software, but no other asset types. The mechanism is asymmetric. Plant and machinery (including computers and software) is scrapped when output falls, but there is no corresponding effect when output rises. Since there have been no occasions during our sample period when non-manufacturing output has suffered an absolute fall on an annual basis, cyclical scrapping affects only manufacturing. This mechanism makes the time path of the capital stock more cyclical.

Chart 21

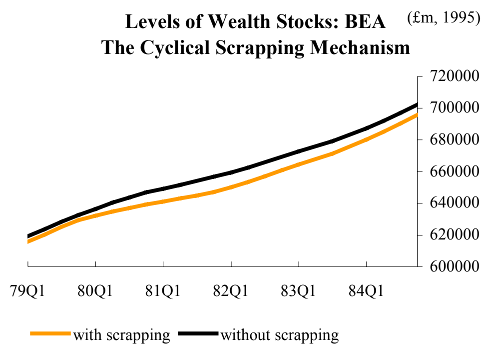


Chart 22

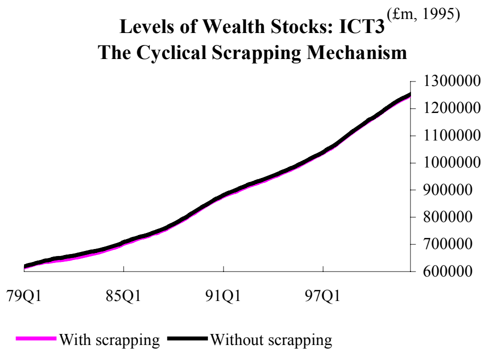


Chart 23a

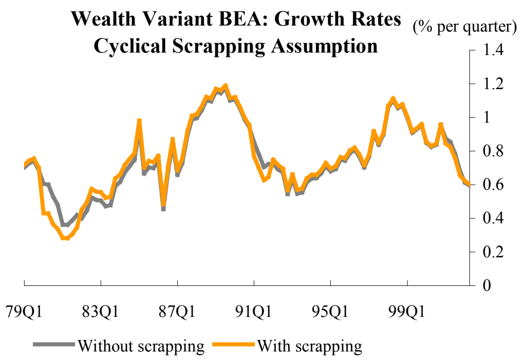


Chart 23b

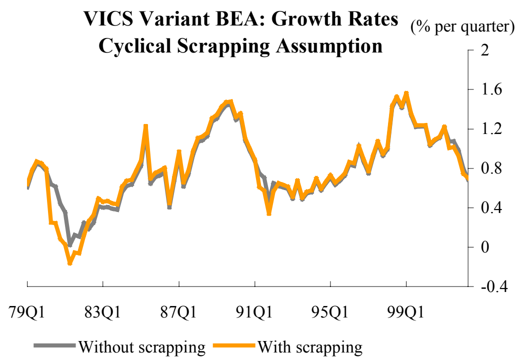


Chart 24a

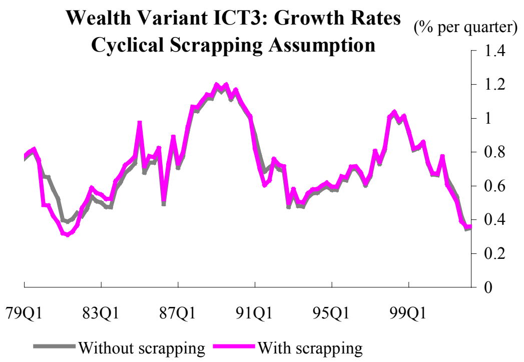
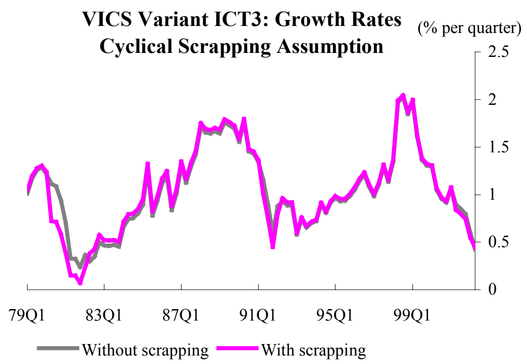


Chart 24b



One could argue that the cyclical scrapping of plant could show up as a shorter life length (and thus higher depreciation rate) in the data in years when it occurs. In other words, there is no need to adjust the stock of plant in manufacturing for cyclical scrapping because the depreciation rate applied to plant in those years already embodies the effect. This could be the case if the depreciation rate used for plant was time varying. However, we use a constant depreciation rate which we assume is the rate in 'normal' years. Hence we add on the mechanism described earlier in this section for certain years to adjust the depreciation rate for cyclical scrapping.

The impact on the level of the wealth stock is minor, since by construction the mechanism is temporary (Charts 21 and 22). There is a similar pattern to the level of the VICS measures. The growth rates of wealth (Charts 23a and 24a) and VICS (Charts 23b and 24b) become more pronounced in downturns (the recession years 1979 and 1991) but catch up in upturns. So there is only a small effect on the overall level by the end of the period.

Estimates of aggregate depreciation

Aggregate depreciation (capital consumption) is the difference between gross and net domestic product. In the past, net domestic product (NDP) has received much less analytical attention than gross. But Weitzman (1976) argued that net, not gross, domestic product is the appropriate measure of welfare.(45) And King (2001) has argued that GDP may be a misleading measure of output when the mix of assets is shifting towards shorter-lived ones, a situation where aggregate depreciation may be rising. This subsection presents estimates of the aggregate nominal depreciation rate and depreciation (as a percentage of GDP) for the six variants. We use the nominal measure because it has a natural interpretation. Section 3 showed that the aggregate nominal depreciation rate can be thought of as a weighted average of the depreciation rates on individual assets, where the weights are the shares of each asset in aggregate nominal wealth.

Variants BEA and ONS1 were constructed on the basis of different depreciation rates at the asset level (for plant and buildings). Variant BEA has the higher US rates while ONS1 has lower rates, corresponding to the longer service lives assumed by the ONS. The discussion of straight-line depreciation in conclusion to Section 3, noted that, if the depreciation rates used to estimate the UK stocks were raised to US levels, then the aggregate depreciation rate would be raised. This is evident from Chart 25. The nominal aggregate depreciation rate on the BEA variant is almost twice that of the ONS1 variant. However, while there is a large difference in the levels of the aggregate rate, the two variants show a similar pattern of variation over time.

Chart 26 shows a comparison of nominal depreciation rates between the United Kingdom and the United States.(46) The US rate has risen fairly steadily over two decades, standing now at about 9% per annum. Unlike in the United States, and despite its deliberate methodological similarity, in the UK variant BEA does not show any tendency to increase substantially in the 1990s (nor does variant ONS1 as is clear from Chart 25). The reason is that buildings (which have the largest share in nominal wealth) continued to dominate in this period. Plant grew faster than buildings (compare Charts E.1 and E.2 in Appendix E) but its price (relative to buildings) fell so that its share in the total did not increase. Variant ICT3 is the closest methodologically to the US NIPA. It shows a slightly larger rise in the 1990s than does variant BEA (see also Chart 27) but

(45) In Weitzman's concept, NDP is deflated by the price index of consumption. Weitzman's concept is discussed further in Oulton (2002).
(46) The UK coverage is whole economy less dwellings and the US coverage is the private non-residential sector.

still does not match the upward trend in the United States. This is because the faster growing assets experienced large price falls and their shares in nominal wealth were very small. To replicate the US nominal depreciation rate experience, the shares of computers and software in the UK wealth stocks would need to be higher than they are at present.

Chart 28 presents depreciation as a proportion of GDP (in current prices) for all the measures.(47) It also includes the ratio calculated from ONS data (labelled 'official'). The 'official' series appears to have a downward trend.

It is noteworthy that even though the nominal aggregate depreciation rate shows an upward trend in the United States, depreciation as a proportion of GDP is almost flat (Chart 29) up to around 1999. But both variant ICT3 for the UK and the US ratio show an uptick in the last couple of years; this may be partly due to cyclical factors.

The relative constancy of the depreciation to GDP ratio can be explained by making use of the following identity:

$$\textit{Depreciation/GDP} = \textit{(Depreciation/Capital Stock)} \times \textit{(Capital Stock/GDP)}$$

In the United States the capital stock to GDP ratio (in current prices) has moved in the opposite direction to the depreciation rate so the depreciation to GDP ratio is relatively flat. In the United Kingdom, for variants BEA and ONS1, the depreciation rate has fallen slightly (Chart 25), while the capital stock to GDP ratio has remained fairly stable. Hence the depreciation to GDP ratios for these variants has fallen slightly. For variant ICT3, in the latter part of the period, the depreciation rate has risen as has the capital stock to GDP ratio. These effects reinforce each other so that the depreciation to GDP ratio rises.

---

(47) GDP in current prices (ONS code: ABML) includes an estimate of capital consumption (ONS code: NQAE). To get the ratio of nominal depreciation to GDP consistent, we first subtract the ONS estimate of capital consumption from the denominator of the ratio and then add back our measure of depreciation. This way the depreciation figures in the numerator and denominator are consistent. Note that NQAE is inclusive of dwellings, whereas our measures are for whole economy minus dwellings. So when calculating the ONS ratio we make a further adjustment for capital consumption in dwellings (ONS code: EXCT). For example, the depreciation GDP ratio for Variant ICT3 is DEPICT3/(ABML - (NQAE - EXCT) + DEPICT3) and for the ONS ratio it is (NQAE-EXCT)/ABML.

Chart 25

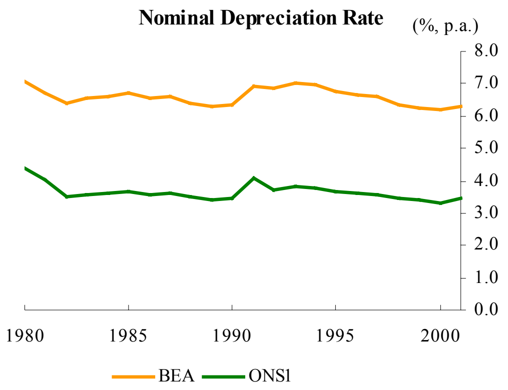


Chart 26

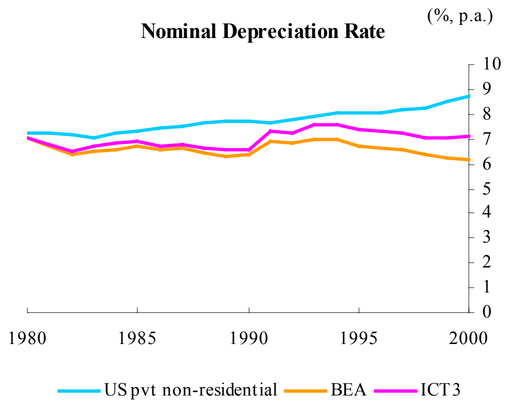


Chart 27

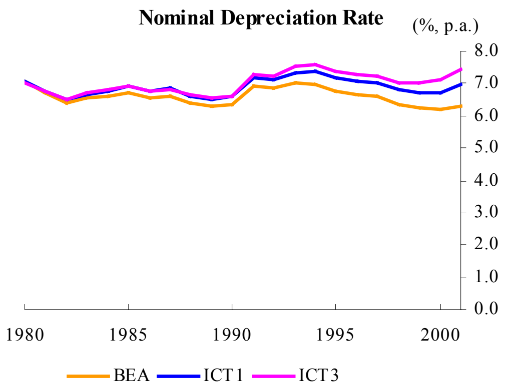


Chart 28

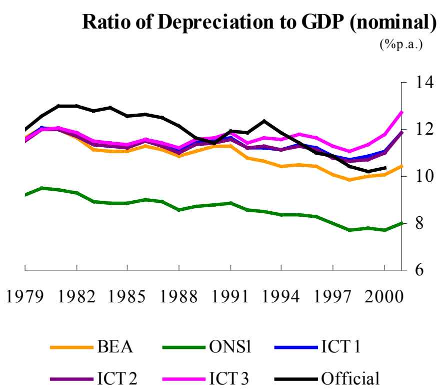
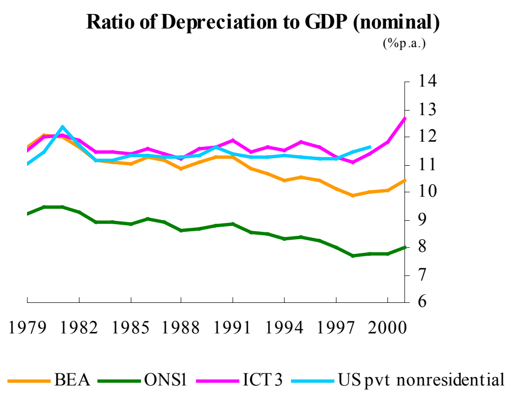


Chart 29



Does cyclical scrapping affect the results qualitatively? The answer is no. As is evident from Charts 30-32 which show the aggregate depreciation rate for variant ICT3, the temporary nature of cyclical scrapping affects the aggregate rate (nominal or real) for only a very short time. The same holds true for the other variants.

Chart 30

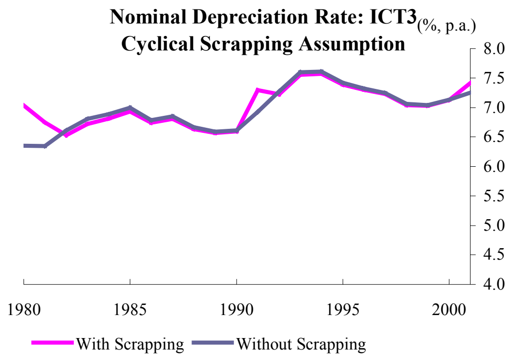


Chart 31

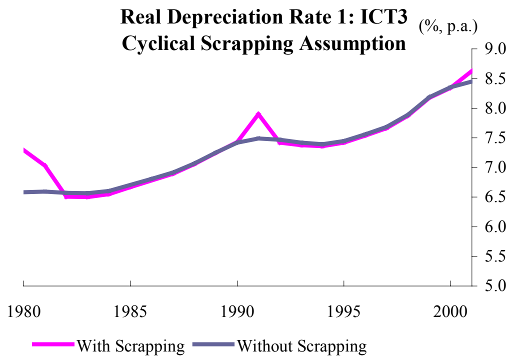
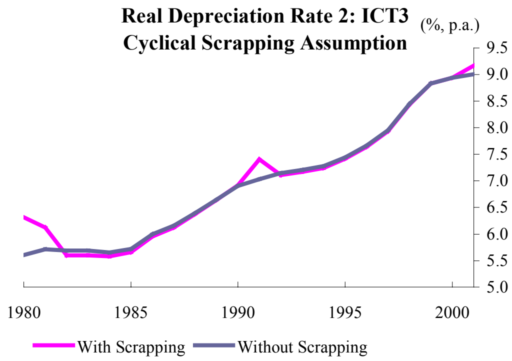
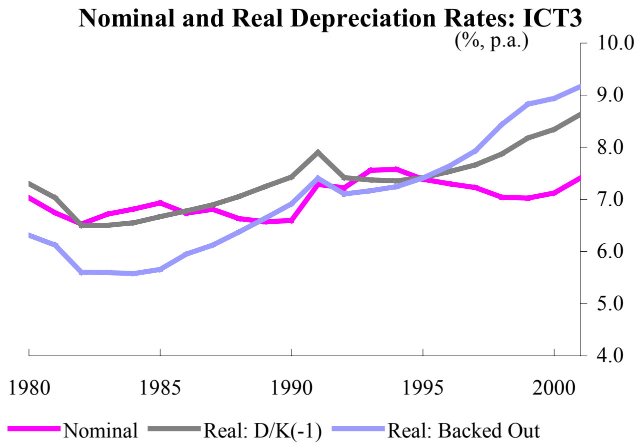


Chart 32



Note that the overall pattern of the real depreciation rate is different in Charts 31 and 32 depending on the method used to calculate it. Section 3 discussed two methods of calculating the real aggregate depreciation rate. Real measure one calculated it as a ratio of real depreciation to real capital stock and real measure two backed it out from the capital accumulation identity equation. Because we use chain-linked data, there is no reason why real measure one should equal real measure two. Chart 33 presents nominal and both real measures of aggregate depreciation rate for variant ICT3. The caveat stated in the conclusion to the subsection on aggregate depreciation in Section 3 should be kept in mind: we should use the real definitions of the aggregate depreciation rate with caution.

Chart 33



Profitability

In the process of estimating the VICS, we have had to calculate the nominal, post corporation tax, rate of return on capital for a range of assumptions about depreciation rates and asset composition. This rate of return has some independent interest as a measure of profitability. So we would like to know how sensitive it is to our assumptions.

The nominal rate of return is defined as aggregate nominal profit, net of depreciation and net of corporation tax, as a percentage of the nominal value of the aggregate capital stock at the beginning of the year. To recall: all fixed assets (excluding dwellings) plus inventories are included in the aggregate capital stock when the rate of return is estimated and the stock itself is net of depreciation. We can convert this rate of return into a real rate by subtracting from it a measure of inflation. For the latter we use the growth rate of the GDP deflator.[48]

Charts 34 and 35 show the nominal and real rates for three sets of assumptions: BEA, ONS1 and ICT3. The rates of return turn out to be remarkably insensitive to the depreciation rate assumptions. The explanation is that, while a higher depreciation rate raises the amount of depreciation on a given stock of an asset, thus lowering net profit, it simultaneously reduces the estimated stock. It turns out that these two effects roughly cancel out. In fact, the rate of return is lowest with the low depreciation rates (and long asset lives) of the ONS1 assumption. Over the most recent period 1995-2000, the real rate averaged 6.67% per annum under ICT3, 7.04% under BEA and 5.90% under ONS1.

The real rate of return is highly cyclical, falling sharply in the two major recessions of 1980-82 and 1990-92 and even turning negative in the latter. Over 1979-2000, it averaged 4.0% to 4.7% per annum (depending on the assumptions). If this appears low, recall that our measure is for the whole economy and so includes health, education and government, sectors where profit is not the main concern.

[48] The GDP deflator is gross value added at current basic prices divided by gross value added at 1995 basic prices (ONS codes ABML ÷ ABMM).

Chart 34

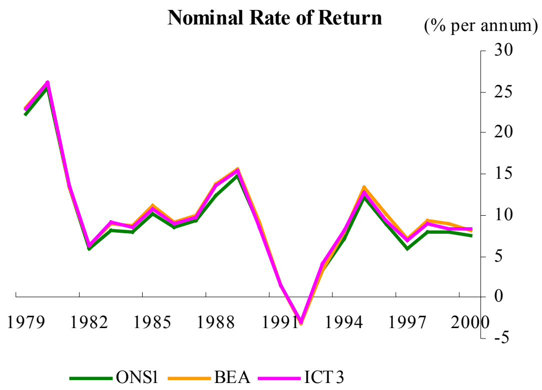
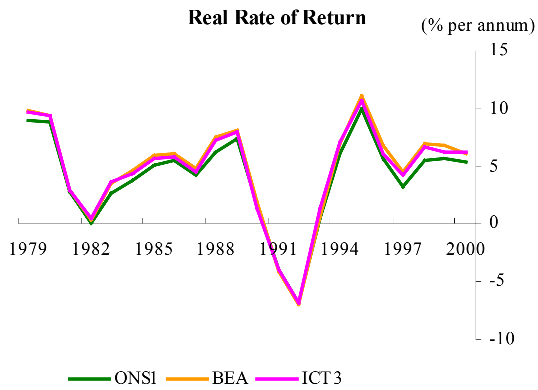


Chart 35



5 Conclusions

We have set out an integrated framework for estimating the wealth stock, the VICS, and depreciation (capital consumption). The resulting estimates are consistent both theoretically and empirically. In this framework the distinction between decay, which describes how the services of a capital asset change as the asset ages, and depreciation, which describes how the prices of assets of different ages vary, is crucial. We have seen that the estimation process is greatly simplified if we adopt the assumption that depreciation is geometric, since then the rates of decay and depreciation are equal. We have reviewed the evidence for geometric depreciation. Unfortunately, there is little direct evidence for the United Kingdom. Most studies relate to the United States and even here the evidence is far from complete. But it is fair to say that the geometric assumption is found to fit the facts quite well. Hence, it has been officially adopted as the 'default' assumption in the US NIPA.

The paper has also considered whether the geometric assumption is appropriate for assets like computers. Computers do not suffer much from physical wear and tear, but nevertheless have very short lives due to what is usually called 'obsolescence'. We found that, in principle, our framework encompasses obsolescence. A properly specified hedonic regression, applied to panel data on new and second-hand asset prices, can estimate the true rate of depreciation, even in the presence of obsolescence. But if an empirically important quality variable is omitted from the regression, the estimate of depreciation will be biased. If quality is improving the bias will be positive, ie the estimated rate of depreciation will be too high. The depreciation rates used by the BEA in the US NIPA are based on studies of new and second-hand asset prices. Since these studies were not always able to control fully for (generally rising) quality, it may be that the BEA rates are overstated. In the specific case of computers, we have argued that the US evidence supports a geometric rate of about 30% rather than the 40% used by the BEA.

We have accordingly adopted the geometric assumption in our empirical work for the United Kingdom. Because of the uncertainty about asset lives and the pattern of depreciation in the United Kingdom, we have calculated wealth and VICS measures under a range of assumptions. We have tested the sensitivity of our results in three main ways. First, we compare results using both US and UK assumptions about asset lives. Second, we compare results based on a comparatively coarse breakdown of assets into four types only, with results derived from a more detailed breakdown in which computers and software are distinguished separately. Third, we compare the effect of US versus UK price indices for computers and software. Our results are for the whole economy and all fixed assets excluding dwellings, for the period 1979 Q1-2002 Q2. Our main findings for wealth and VICS are as follows:

1. Using the conventional, four fold breakdown of assets into buildings (excluding dwellings), plant and machinery, vehicles, and intangibles, we find that the growth rates of wealth and the VICS are insensitive to variations in depreciation rates.
2. By the nature of the measure, the level of the VICS will be insensitive to depreciation rates if the growth rate is. This is because the real level of the VICS equals the nominal level in the base year, whatever the assumption about depreciation. This nominal level is just aggregate profit in current prices. So before and after the base year the estimated level will be affected by different assumptions about depreciation only to the extent that the growth rates are affected. However, no such restriction applies to wealth. In fact, the *level* of wealth is found to be quite sensitive to variations in depreciation rates.
3. Still sticking with the conventional asset breakdown, wealth and VICS grew at similar rates over the period as a whole. In the 1990s, the gap between the two measures widens a bit, with the growth rate of the VICS higher by about 0.1 percentage points per quarter.
4. The effect on the estimates of separating out computers and software is quite complex. First, with these assets separated out, much larger differences appear between the growth rates of VICS and wealth, of the order of 0.2-0.4 percentage points per quarter. Second, comparing results with and without computers and software being separated, we find that separating them out tends to reduce the growth rate of wealth, while not necessarily increasing that of the VICS. But when we use the set of assumptions which are closest to US methods (eg US price indices for computers and software plus the 'times 3' adjustment to the level of software investment), the growth rate of the VICS is raised by 0.2 percentage points per quarter, relative to the VICS with computers and software included in with other assets.
5. The VICS tends to be more volatile than wealth when volatility is measured by the standard deviation of the growth rate. Using US methods for computers and software tends to raise volatility.

For some purposes, growth rates are more important than levels. If so, these results suggest that the empirically important issue is the measurement of investment in computers and software. It is common ground that the relative price of these assets has been falling, so in principle it is correct to separate them out explicitly. The conclusions about the growth rates of both VICS and wealth turn out to be very sensitive to the price index used for computers and to the correction made to the level of software investment.

For other purposes, eg measuring Tobin's Q, levels matter. In these cases, there is no substitute for further research into asset lives. But it turns out that profitability, the real rate of return on capital, is not sensitive to the asset life assumptions.

We have also estimated aggregate depreciation (capital consumption) for the same range of assumptions. We have studied the sensitivity of the aggregate depreciation rate and of the ratio of depreciation to GDP to the assumptions, and compared our estimates with ones derived from official data. On theoretical grounds we prefer to measure both these ratios in current prices. Our findings here are as follows:

1. Using the conventional asset breakdown and our assumptions about depreciation rates at the asset level, there is no tendency for the aggregate depreciation rate to rise over the last two decades. In other words, the asset mix has not been shifting towards more rapidly depreciating assets like plant and machinery, vehicles or intangibles.
2. Separating out computers and software has less effect than one might have expected. Even using US methodology raises the aggregate rate by only about 1 percentage point to 7% in 2000 and again there is no sign of an upward trend. The reason is that even by 2000 the share of computers and software in wealth was only about 4%. By contrast and on a comparable basis, the aggregate depreciation rate in the United States has trended smoothly upwards since 1980, to reach nearly 9% in 2000. This illustrates the much greater scale of ICT investment in the United States.
3. The assumptions about asset lives have a large impact on the estimated ratio of depreciation to GDP. The official measure taken from the national accounts has been drifting down fairly steadily since 1979. In 2001 it stood at 8%. Using US asset lives and the conventional asset breakdown, the ratio was over 10% in the same year. Separating out ICT assets and using US methods, the ratio rises to nearly 13%, similar to the ratio in the United States. Interestingly, in neither country is there any upward trend in the ratio, except perhaps in the past couple of years. The reason is that though the quantity of high-depreciation assets has been growing faster than GDP, this has been offset by their falling price.

References

Australian Bureau of Statistics (2001), *Australian national accounts: concepts, sources and methods*; available from www.abs.gov.au.

Berndt, E R and Fuss, M A (1986), 'Productivity measurement with adjustments for variations in capacity utilisation and other forms of temporary equilibrium', *Journal of Econometrics*, Vol. 33, pages 7-29.

Central Statistical Office (1985), *United Kingdom National Accounts: Sources and Methods*, 2nd edition, London: HMSO.

Chow, G C and Lin, A (1971), 'Best linear unbiased interpolation, distribution and extrapolation of time series by related series', *Review of Economics and Statistics*, Vol. 53, pages 372-75.

Cummins, J G and Violante, G L (2002), 'Investment-specific technical change in the U.S. (1947-2000): measurement and macroeconomic consequences', Washington, D.C.: Board of Governors of the Federal Reserve System.

Dean, G (1964), 'The stock of fixed assets in the United Kingdom in 1961', *Journal of the Royal Statistical Society*, Series A, Vol. 127, pages 327-52.

Diewert, W E (1980), 'Aggregation problems in the measurement of capital', in Usher, D (ed), *The measurement of capital*, Studies in Income and Wealth No. 45, pages 433-528, Chicago: The University of Chicago Press.

Diewert, W E and Lawrence, D A (2000), 'Progress in measuring the price and quantity of capital', in Lau, L J (ed), *Econometrics volume 2: econometrics and the cost of capital: essays in honour of Dale W Jorgenson*, Cambridge, MA: The MIT Press.

Feldstein, M and Rothschild, M (1974), 'Towards an economic theory of replacement investment', *Econometrica*, Vol. 42, pages 393-424.

Feller, W (1968), *Introduction to probability theory and its applications* (3rd edition), J Wiley and Sons.

Fisher, F M (1965), 'Embodied technical change and the existence of an aggregate capital stock', *Review of Economic Studies*, Vol. 32, pages 263-88.

Fraumeni, B M (1997), 'The measurement of depreciation in the U.S. national income and product accounts', *Survey of Current Business*, (July), pages 7-23.

Gordon, R J (1990), *The measurement of durable goods prices,* Chicago, University of Chicago Press.

Griffin, T (1976), 'The stock of fixed assets in the United Kingdom: how to make the best use of the statistics', *Economics Trends,* No. 276 (October), pages 130-43.

Herman, S W (2000), 'Fixed assets and consumer durable goods', *Survey of Current Business*, April, pages 17-30.

Hall, R E (2001), 'The stock market and capital accumulation', *American Economic Review*, Vol. 91, pages 1,185-202.

Hall, R E and Jorgenson, D W (1967), 'Tax policy and investment behaviour', *American Economic Review*, Vol. 57, pages 391-414.

Hulten, C R and Wykoff, F C (1981a), 'The estimate of economic depreciation using vintage asset prices', *Journal of Econometrics*, Vol. 15, pages 367-96.

Hulten, C R and Wykoff, F C (1981b), 'The measurement of economic depreciation', in Hulten, C R (ed), *Depreciation, inflation and the taxation of income from capital*, Washington, D.C.: The Urban Institute Press.

Hulten, C R and Wykoff, F C (1996), 'Issues in the measurement of economic depreciation: introductory remarks', *Economic Inquiry,* Vol. XXXIV (January), pages 57-77.

Inland Revenue (1953), *Wear and tear allowances for machinery and plant: list of percentage rates*, London: HMSO.

Jorgenson, D W (1989), 'Capital as a factor of production', in Jorgenson, D W, and Landau, R (eds), *Technology and capital formation*, Cambridge, MA: The MIT Press.

Jorgenson, D W (1996), 'Empirical studies of depreciation', *Economic Inquiry*, Vol. 24, pages 24-42.

Jorgenson, D W, Gollop, F M and Fraumeni, B M (1987), *Productivity and U.S. economic growth*, Cambridge, MA.: Harvard University Press.

Jorgenson, D W, and Griliches, Z (1967), 'The explanation of productivity change', *Review of Economic Studies*, Vol. 34, pages 249-83; reprinted in D W Jorgenson, *Productivity: volume 1: postwar U.S. economic growth,* Cambridge, MA: The MIT Press.

Jorgenson, D W, and Stiroh, K J (2000), 'Raising the speed limit: U.S. economic growth in the information age', *Brookings Papers on Economic Activity,* Vol. 1, pages 125-211.

King, M (2001), ''Hanes dwy ddinas' or 'A tale of two cities'', speech to Cardiff Business School, Cardiff University, 18 June 2001, reprinted in *Bank of England Quarterly Bulletin*, Autumn, pages 336-39.

Lane, R N (1999), *Appraisal report, 'large aerospace firm', personal property, Los Angeles County, March 1, 1995 (revised February 2, 1999)*, Lane, Westly Inc.: Burlinghame, CA.

Larsen, J, Neiss, K and Shortall, F (2002), 'Factor utilisation and productivity estimates for the United Kingdom', *Bank of England Working Paper no. 162.*

Mayes, D and Young, G (1994), 'Improving the estimates of capital stock', *National Institute Economic Review*, No. 147 (February), pages 84-96.

OECD (2001a), *OECD productivity manual: a guide to the measurement of industry-level and aggregate productivity growth*, Paris: OECD.

OECD (2001b), *Measuring capital: a manual on the measurement of capital stocks, consumption of fixed capital and capital services,* Paris: OECD.

Office for National Statistics (1998), *United Kingdom National Accounts Concepts, Sources and Methods*, 1998 edition, London: The Stationery Office.

Oliner, S D (1993), 'Constant-quality price change, depreciation, and retirement of mainframe computers', in Foss, M F, Manser M E, and Young AH (eds), *Price measurements and their uses*, NBER Studies in Income and Wealth Number 57, Chicago: Chicago University Press.

Oliner, S D (1994), 'Measuring stocks of computer peripheral equipment: theory and application', Board of Governors of the Federal Reserve System, *mimeo*.

Oliner, S D (1996), 'New evidence on the retirement and depreciation of machine tools', *Economic Inquiry*, Vol. XXXIV, pages 57-77.

O'Mahony, M (1999), *Britain's productivity performance 1950-1996: an international perspective*, London: National Institute of Economic and Social Research.

Oulton, N (1995), 'Depreciation, obsolescence and the role of capital in growth accounting', *Bulletin of Economic Research*, Vol. 47, pages 21-34.

Oulton, N (2001a), 'ICT and productivity growth in the United Kingdom', *Bank of England Working Paper no. 140*.

Oulton, N (2001b), 'Measuring capital services in the United Kingdom', *Bank of England Quarterly Bulletin*, Autumn, pages 295-309.

Oulton, N (2002), 'Productivity versus welfare: or, GDP versus Weitzman's NDP', *Bank of England Working Paper no. 163*.

Oulton, N and O'Mahony, M (1994), *Productivity and growth: a study of British industry 1954-1986*, Cambridge: Cambridge University Press.

Parker, R P and Grimm, B (2000), 'Recognition of business and government expenditures for software as investment: methodology and quantitative impacts, 1959-98', Washington, D.C.: Bureau of Economic Analysis; available from the BEA website, www.bea.doc.gov/bea/papers/software.pdf

Prucha, I (1997), 'Estimation of a variable rate of depreciation: a dummy variable approach', *Structural Change and Economic Dynamics* Vol. 8, pages 319-25.

Redfern, P (1955), 'Net investment in fixed assets in the United Kingdom, 1938-55', *Journal of the Royal Statistical Society*, Series A, Vol. 118, pages 141-82.

Smith, A D (1986), 'The feasibility of fire insurance estimates of capital stock'. *National Institute of Economic and Social Research Discussion Paper (Old Series) no. 116.* London.

U.S. Department of Commerce (1999), *Fixed reproducible tangible wealth in the United States, 1925-94,* Washington, DC: U.S. Government Printing Office, available at http://www.bea.doc.gov.

Wadhwani, S and Wall, M (1986), 'The UK capital stock — new estimates of premature scrapping', *Oxford Review of Economic Policy*, Vol. 2, no. 3, pages 44-55.

Weitzman, M L (1976), 'On the welfare significance of national product in a dynamic economy', *Quarterly Journal of Economics*, Vol. XC, pages 156-62.

West, P and Clifton-Fearnside, A (1999), 'The capital stock of the United Kingdom — some new developments in coverage and methodology', *Economic Trends*, No. 554 (March), pages 39-41.

Whelan, K (2000a), 'Computers, obsolescence, and productivity', Washington, D.C.: Board of Governors of the Federal Reserve.

Whelan, K (2000b), 'A guide to the use of chain aggregated NIPA data', Washington, D.C.: Board of Governors of the Federal Reserve.

Estimates to accompany Working Paper No. 192
http://www.bankofengland.co.uk/archive/Documents/historicpubs/workingpapers/2003/CAPDATA.xls

Appendix A: Proofs of propositions in the text

A.1. Proof that geometric depreciation implies geometric decay and of the converse

The price of a new asset at the end of period t-1 is:

$$p^A_{t-1,0} = \sum_{z=0}^{\infty}\left[p^K_{t+z,z} \Big/ \prod_{\tau=0}^{z}(1+r_{t+\tau})\right] \quad \textbf{(49)}$$

This is the same as equation **(2)** of the main text except that we have made the possibility of an infinite life explicit. Analogously, the price of an asset of age s at the same time is:

$$p^A_{t-1,s} = \sum_{z=0}^{\infty}\left[p^K_{t+z,s+z} \Big/ \prod_{\tau=0}^{z}(1+r_{t+\tau})\right] \quad \textbf{(50)}$$

The relationship between the rental prices is given by **(14)**, repeated here for convenience:

$$p^K_{t,s} / p^K_{t,0} = f_s / f_0$$

Similarly,

$$p^K_{t+z,s+z} / p^K_{t+z,z} = f_{s+z} / f_z, \qquad z = 0,1,2,...$$

Dividing **(50)** by **(49)**, the ratio of the two asset prices is:

$$p^A_{t-1,s} / p^A_{t-1,0} = \sum_{z=0}^{\infty}\left[(f_{s+z}/f_z)\cdot p^K_{t+z,z}\cdot h_z\right] / \sum_{z=0}^{\infty}\left[p^K_{t+z,z}\cdot h_z\right] \quad \textbf{(51)}$$

where we have put:

$$h_z = 1 / \prod_{\tau=0}^{z}(1+r_{t+\tau})$$

for the discount factor.

Geometric decay implies geometric depreciation

If decay is geometric, then $f_{s+z} / f_z = (1-d)^s$. That is, the decay factor depends only on the difference in the ages, not the absolute ages. The ratio of the asset prices is then:

$$p^A_{t-1,s} / p^A_{t-1,0} = (1-d)^s$$

since we can factor the decay factor out from the summation in **(51)**. Hence depreciation is at the geometric rate d, ie $\delta = d$.

Geometric depreciation implies geometric decay

If depreciation is geometric at rate δ, then from **(51)**

$$(1-\delta)^s = \sum_{z=0}^{\infty}\left[(f_{s+z}/f_z)\cdot p^K_{t+z,z}\cdot h_z\right] / \sum_{z=0}^{\infty}\left[p^K_{t+z,z}\cdot h_z\right], \qquad s = 0,1,2,...$$

The only term on the right-hand side of this equation which depends on s is the factor f_{s+z} / f_z. Hence the only way that this equation can be satisfied for all values of s is if f_{s+z} / f_z is independent of the absolute ages and depends only on the difference in ages, so that we can write $f_{s+z} / f_z = \phi_s$ say. Then we can factor it out of the summation to obtain:

$$(1-\delta)^s = \phi_s = (1-d)^s$$

Hence decay is at the geometric rate δ, ie $d = \delta$.

A.2. Proof that assets with proportionally high rental prices receive more weight in a VICS than in a wealth measure

According to equation **(28)**, the relation between rental prices and asset prices is:

$$p_{it}^K = T_{it}\left[r_t \cdot p_{i,t-1}^A + \delta_i \cdot p_{it}^A - (p_{it}^A - p_{i,t-1}^A)\right]$$

$$= T_{it}\left[r_t + \delta_i(1+\pi_{it}) - \pi_{it}\right] p_{i,t-1}^A$$

where $\pi_{it} = (p_{it}^A - p_{i,t-1}^A) / p_{i,t-1}^A$, the rate of growth of the asset price. Now define the ratio of the rental price to the asset price by:

$$\rho_{it} = p_{it}^K / p_{i,t-1}^A$$

To simplify notation, normalise all asset prices to unity: $p_{i,t-1}^A = 1, \forall i$. The VICS weights are now:

$$w_{it} = \rho_{it} K_{it} / \sum_{i=1}^m \rho_{it} K_{it}$$

and the weights in the wealth measure are:

$$v_{i,t-1} = K_{it} / \sum_{i=1}^m K_{it}$$

Then we have the following:

Proposition If asset *i* has a rental asset price ratio which is higher than the asset-value-weighted average for all assets, then its weight in the VICS is higher than its weight in the wealth measure. In symbols,

$$\text{If } \rho_{it} > \sum_{j=1}^m v_{jt}\rho_{jt}, \text{ then } w_{it} > v_{i,t-1}$$

Proof Assume the contrary: $w_{it} \le v_{i,t-1}$. Then from the definition of the weights:

$$\frac{\rho_{it} K_{it}}{\sum_{j=1}^{m} \rho_{jt} K_{jt}} \leq \frac{K_{it}}{\sum_{j=1}^{m} K_{jt}}$$

which implies that:

$$\rho_{it} \leq \sum_{j=1}^{m} \left(\frac{K_{jt}}{\sum_{j=1}^{m} K_{jt}} \right) \cdot \rho_{jt} = \sum_{j=1}^{m} v_{jt} \rho_{jt}$$

This is a contradiction, so the proposition is proved.

A.3 Proof of proposition about real depreciation rate, δ_t^R

Proposition

Suppose as in the text that there are two assets. Both assets are growing at constant rates but the first asset is growing more rapidly and also has a higher depreciation rate. Technology is Cobb-Douglas, so that the current price share of each asset in the aggregate capital stock is constant. Consider the ratio of real depreciation to the real capital stock. Then (a) under chain-linking, this ratio will rise without limit so that eventually it exceeds the rate on the higher of the two individual depreciation rates; (b) with a fixed base index, the ratio approaches the higher of the two rates asymptotically.

Proof

(a) Chain-linking

This result will be proved using a Törnqvist chain index, which is generally a good approximation to a Fisher chain index. Let D_{it} be real depreciation on asset type *i* in period *t*:

$$D_{it} = \delta_i A_{i,t-1}$$

The Törnqvist chain index of aggregate real depreciation is:

$$\Delta \ln D_t = \sum_{i=1}^{m} w_i \Delta \ln D_{it} = \sum_{i=1}^{m} w_i \Delta \ln A_{i,t-1}$$

where:

$$w_i = \delta_i p_{it} A_{i,t-1} / \sum_{i=1}^{m} \delta_i p_{it} A_{i,t-1}, \text{ all } t.$$

are the constant-over-time wealth shares. The Törnqvist chain index of wealth is:

$$\Delta \ln A_{t-1} = \sum_{i=1}^{m} v_i \Delta \ln A_{i,t-1}$$

where:

$$v_i = p_{it} A_{i,t-1} / \sum_{i=1}^{m} p_{it} A_{i,t-1}, \text{ all } t$$

Now specialise these definitions to the case of two assets. Asset 1 has a high depreciation rate, asset 2 a low one ($\delta_1 > \delta_2$). Suppose that the growth rates of the assets are constant but that asset 1 grows more rapidly: $g_1 > g_2$ where the g_i are the growth rates (defined as log differences). So in this case:

$$\Delta \ln D_t = w_1 g_1 + w_2 g_2 \text{ and } \Delta \ln A_{t-1} = v_1 g_1 + v_2 g_2$$

where $w_1 + w_2 = v_1 + v_2 = 1$. The wealth shares (v_1, v_2) and the shares in aggregate nominal depreciation (w_1, w_2) are related by

$$\frac{w_1}{w_2} = \frac{\delta_1 p_{1t} A_{1,t-1}}{\delta_2 p_{2t} A_{2,t-1}} = \frac{\delta_1}{\delta_2} \frac{v_1}{v_2}$$

Now since by assumption $\delta_1 > \delta_2$, it follows that $w_1 /(1 - w_1) > v_1 /(1 - v_1)$ and so that $w_1 > v_1$. Consequently, $\Delta \ln D_t > \Delta \ln A_{t-1}$. The difference between these two growth rates is constant, so the ratio of real depreciation to real wealth rises without limit. Eventually, the aggregate depreciation rate must exceed the individual rates.

(b) Fixed-base indices

With fixed-base indices, we can set prices in the base year equal to 1, so that aggregate depreciation is:

$$D_t = \delta_1 A_{1,t-1} + \delta_2 A_{2,t-1}$$

and the capital stock is:

$$A_t = A_{1t} + A_{2t}$$

The ratio of depreciation to the capital stock is:

$$\frac{\delta_1 A_{1,t-1} + \delta_2 A_{2,t-1}}{A_{1,t-1} + A_{2,t-1}}$$

If asset 1 is growing faster, then this ratio approaches δ_1 as *t* goes to infinity.

Appendix B: Data appendix

Investment

The following table shows the annual and quarterly, seasonally adjusted investment series we have used, together with the ONS codes for the current and constant price series.

Table B.1 ONS codes for gross investment

	Quarterly series, sa		*Annual series*	
Asset type	*Current prices*	*1995 prices*	*Current prices*	*1995 prices*
1. Other buildings and structures	EQED	DLWT	DLWS[49]	DLWQ
2. Transport equipment	TLPX	DLWL	DLWZ	DLWJ
3. Other machinery and equipment and cultivated assets	TLPW	DLWO	DLXI	DLWM
4. Intangible fixed assets	TLPK	EQDO	DLXP	EQDT
5. Changes in inventories	Not used	CAFU	not used	ABMQ

Real asset stocks

(a) Annual We calculated annual asset stocks from 1963 onwards, using starting stocks for end-1962 generated as in earlier work (Oulton (2001a)) and employing equations **(31)** and **(32)**. The stock of inventories used the value at the end of 2000 in 1995 prices (from the Quarterly National Accounts, 2nd quarter 2001) as the basis. The stock in other years was then calculated from the changes in inventories series.

(b) Quarterly The quarterly investment series start in 1965 Q1. We used the annual capital stock model to generate a starting stock for each asset at the end of 1964 Q4. Then for the four fixed assets, the stock of each asset was accumulated from 1965 Q1 onwards using the quarterly investment series (see above), employing equation **(26)**. The quarterly stock of inventories was calculated in the same way as the annual stock.

The depreciation rates are based on those used by the BEA in the US NIPA, described in Fraumeni (1997), and are shown in Table B.2. The BEA rates themselves are at a more disaggregated level; the rates in the table are averages of these more detailed rates. The average rate for plant and machinery in the United States is now considerably higher than 13%, due to the rise in importance of computers and software; the 13% figure was appropriate for the 1970s. But since later we make special provision for computers and software, the 13% figure has been retained.

(49) This current price annual investment series for buildings does not include transfer costs. In our calculations we have added transfer costs [DFBH] to this series for 1965-98. From 1999 onwards, the annual values are calculated as the sum of the quarterly values for that year. The quarterly series in current and constant prices and the annual series in constant prices for buildings investment include transfer costs already.

Table B.2 Depreciation rates (per cent per annum)

Asset	BEA lives (Variant BEA)	ONS lives (Variant ONS1)
1. Other buildings and structures	2.5	100*0.90/79 = 1.14
2. Other machinery and equipment and cultivated assets	13.0	100*1.65/29 = 5.69
3. Transport equipment	25.0	100*1.853/9= 20.59
4. Intangible fixed assets	31.5	31.5
5. Inventories	0.0	0.0

Asset prices

The asset price of each asset type except inventories is derived as an implicit deflator: the current price investment series divided by the constant price investment series. For inventories, we used the price index for all manufacturing, excluding duties [PNVQ], from 1974 onwards and, prior to then, the price index including duties [PLLU]. The annual asset prices formed part of the estimation of the rental price weights. The annual or quarterly asset prices can also be used to convert asset stocks to nominal terms.

Tax/subsidy factor

The tax/subsidy factors (T_{it}) were kindly supplied by Rod Whittaker (HMT). They are annual. There are separate factors for plant and machinery, industrial buildings, and vehicles. We used the tax factor for plant and machinery for intangibles, computers and software as well. The tax factor for inventories was set equal to 1.

Rental prices

To calculate the rental prices and hence the weights for each asset type in the VICS, we include inventories and the fixed assets and use these to solve for first, the nominal rate of return, and next, for the rental prices. Because dwellings are excluded, the appropriate profit total is the aggregate gross operating surplus minus what should be attributed to ownership of dwellings. Total profit is therefore measured as gross operating surplus [ABNF] less actual and imputed rentals on housing [ADFT+ADFU].

The estimated rental price weights were unsatisfactory in a number of ways. First, the rental weight for buildings plunged in 1974 and 1980 in an implausible manner, while that for plant and machinery rose sharply. These spikes were removed by making 1974 the average of 1973 and 1975, and 1980 the average of 1979 and 1981, for these assets. Second, the rental weight for inventories was extremely volatile. This was dealt with by fitting a time trend to the weight and substituting the predicted for the actual values. The weights were then adjusted so that they continued to sum to 1. Finally, we took a two-year moving average of the weights.

Appendix C: A software investment series for the United Kingdom

This appendix updates the series for software investment presented in Oulton (2001a).[(50)] The nominal series which we use for variant ICT3 is constructed from official data but is then multiplied by three for reasons discussed in Oulton (2001a): this is referred to as the 'times 3' adjustment. The nominal data are deflated by the US price index for pre-packaged software as published in Parker and Grimm (2000), adjusted for changes in the sterling-dollar exchange rate. Pre-packaged software is about a third of the total in the United States, the other two components being custom and own account software. The pre-packaged component is the only one for which a true price index exists. Hence we use this to deflate all software.

C.1 Revising the existing current-price series for software investment

Total software investment in current prices is available from the Supply and Use Tables published by the ONS for 1989-2000. Oulton (2001a) extrapolated this data series backwards to 1964 using information from various input-output tables.

The ONS publish investment series for five categories of assets: dwellings, other buildings and structures, other machinery and equipment (OME), vehicles and intangibles. If we want to treat software as a separate category then we need to know in which of the above five categories it is nested so that we can adjust the figures accordingly to avoid double counting. Despite what a casual reading of Tables 6.2 and 6.3 and paragraph 6.15 of *Concepts, Sources and Methods* might suggest, only part of software investment is included in intangible investment; the rest is in the 'other machinery and equipment' category. So aggregate data for both other machinery and equipment and intangible assets have to be adjusted to avoid double counting.

Using data kindly supplied by the ONS (for 1970 onwards), we can extract a series for that part of software investment that is in the intangible asset category. The part of software investment in other machinery and equipment can then be calculated as a residual. In the calculations, we first calculate software (OME) for 1989-2000 by subtracting software (intangibles; NPJG) from the total. For 1970-88, software (OME) is calculated as a proportion of software (intangibles) where the proportion is that of software (OME) divided by software (intangibles) in 1989. The total for 1970-88 is then calculated as the sum of software (OME) and software (intangibles).[(51)]

C.2 Updating the current-price series for software investment to 2001

The raw data only go so far as 2000 but we require an extended series to 2001. The total and software (intangibles) are extended by assuming that the growth rates in 2000 are the same as that of plant and machinery. Software (OME) is then calculated as the residual.
The annual nominal data for 1970-2001 are converted to quarterly nominal data using the procedure described below. For 2002 Q1-2002 Q2, the quarterly nominal series are extrapolated using the quarterly growth rate of plant and machinery.

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(50) Table B.2, page 59, without the 'times 3' adjustment.

(51) In current prices, *software (OME) + software (intangibles) = software (total).* We use the proportion of *software (OME)/software (intangibles)* in 1989 to extrapolate the *software (OME)* series backwards first; the total is then calculated as the sum of the components. This ensures non-negativity of the sub-categories of total software investment.

C.3 Constant-price series for software investment and the associated investment price deflator

The basic procedure is as follows. For each series (computers and software) we start out with annual nominal data on investment, its associated quarterly investment price deflator(52) and an 'indicator' quarterly series that behaves like the economic variable in question.

1. The annual nominal data are converted into quarterly nominal data by using the Chow-Lin interpolation procedure.(53) This procedure requires an input indicator series. It uses the movement of the indicator series to interpolate the quarterly series from the annual investment series.
2. The quarterly nominal series is then deflated by the quarterly investment price deflator to get the quarterly real investment series. Both the ONS and the BEA take the annual price in the base year to be equal to 1, but this annual price is an arithmetic mean of the quarterly prices. This means that the sum of the quarterly real values will not, in general, equal the annual real value (calculated by dividing the annual nominal value by the annual price deflator =1) in the base year. To be consistent with the ONS, we have to ensure base year consistency in the annual values (ie, nominal = real). Hence, the quarterly series are scaled in the ratio of the base year annual nominal value to the base year 'sum of quarterly reals' so that after the scaling the sum of the quarterly real series is equal to the annual nominal value in the base year.
3. The annual real series for years other than the base year are calculated similarly by summing the scaled quarterly real values.

The implication for the implicit price deflators is that they no longer equal the 'published series'; in fact, they are the published series times the scaling factor. This obviously matters for the levels but (i) has no effect on the growth rates and (ii) the scaling factor is very small for the series that we have (for example, using UK computer prices, the scaling factor is 0.003%). The scaling also means that the annual price deflator in the base year (calculated as the arithmetic mean of the quarterly price deflators) is no longer exactly equal to one but the difference is negligibly small. Table C.1 summarises the procedure.

The indicator series used to convert the annual computer and software investment series into their respective quarterly counterparts is the (quarterly) nominal investment in (total) plant and machinery.

(52) If quarterly price deflator series are not available, we have used the annual deflator series and kept the value in each quarter of the year equal to the annual value (eg, done for US computer and software prices).
(53) See Chow and Lin (1971). A RATS subroutine developed by John Frain (Central Bank of Ireland) is used for the interpolation.

Table C.1

	Frequency of Data	
Units of Measurement	Annual Nominal Chow-Lin Procedure using 'Indicator' series	Annual Real
	Quarterly Nominal Quarterly price deflator	Scale and Sum Quarterly Real

Assuming that the deflators are the same across software components (in OME, in intangibles and total), we calculate the constant price series by dividing the current price series by the software investment deflator.

Table C.2

Software Investment in the United Kingdom

	In Intangible Asset Category (£ million, current prices)	*In Other Machinery and Equipment Category* (£ million, current prices)	*Total*	*Implied Deflator*(54)	*In Intangible Asset Category* (constant prices)	*In Other Machinery and Equipment Category* (constant prices)	*Total*(55)
1970	17	12	29	9.77	2	1	3
1971	21	15	36	8.17	3	2	4
1972	24	17	41	7.05	3	2	6
1973	31	22	53	6.97	4	3	8
1974	38	27	65	6.51	6	4	10
1975	49	35	84	6.57	7	5	13
1976	66	48	114	7.29	9	7	16
1977	79	57	136	6.90	11	8	20
1978	99	72	171	5.05	20	14	34
1979	128	93	221	4.09	31	23	54
1980	160	116	276	3.24	49	36	85
1981	182	132	314	3.47	52	38	90
1982	233	169	402	3.75	62	45	107
1983	283	205	488	3.89	73	53	125
1984	345	250	595	3.91	88	64	152
1985	455	329	784	3.64	125	90	215
1986	533	386	919	2.82	189	137	326
1987	591	428	1019	2.28	259	187	446
1988	709	513	1222	1.94	365	264	629
1989	864	625	1489	1.76	491	355	846
1990	1030	781	1811	1.40	734	556	1290
1991	1101	700	1801	1.35	816	520	1336
1992	1180	647	1827	1.08	1091	597	1688
1993	1235	827	2062	1.21	1018	682	1700
1994	1286	1146	2432	1.09	1182	1056	2238
1995	1320	1535	2855	1.00	1320	1535	2855
1996	1548	1483	3031	0.95	1624	1553	3177
1997	1703	1317	3020	0.83	2051	1586	3637
1998	2206	2071	4277	0.76	2910	2733	5643
1999	2417	2016	4433	0.76	3196	2665	5861
2000	2801	1922	4723	0.81	3456	2374	5830
2001	2790	1914	4704	0.83	3351	2300	5651

Note Components in the above tables may not add up to the totals exactly due to rounding.

(54) Because of the scaling required to convert quarterly real data to annual real data, the implied deflator is not exactly equal to sterling equivalent of the Parker and Grimm (2000) annual deflator (conversion from dollars done using the sterling exchange rate (ONS code: AJFA)).
(55) The components add to the total because we are using the same investment price deflator.

Appendix D: Backing out non-computer investment from total investment

D.1 Introduction

The series for investment in plant and machinery published by the ONS includes computers. This appendix considers how to reconstruct the series that ONS would have arrived at, had they decided to exclude computers. For this purpose we use ONS methods and rely entirely on ONS data.

The ONS publishes data on total investment in 'Other machinery and equipment' (*OME*), which includes computers, in both constant and current prices. We also have ONS data on a component of *OME*, computer investment, for the period 1976-2000. The nominal computer series derives from the Input-Output Supply and Use Tables for 1989 onwards; prior to 1989, our series is constructed from the various input-output tables, with missing years interpolated. The real computer series is the nominal series deflated by the official PPI for computers (ONS code PQEK). We want to derive investment in *OME* excluding computers (*OMEXC*). Obviously, there is no problem in doing this in current prices by simple subtraction, but how to do it in constant prices is not so straightforward.

D.2 The chain-linked solution

For the period 1994 to the present, the ONS uses 1995 prices. So for this period we can indeed calculate *OMEXC* by subtracting computer investment in 1995 prices from total *OME* in 1995 prices. But prior to 1994 the ONS used different weights: successively 1990, 1985, 1980 and 1975 prices as we go back in time. In other words the ONS does not use a fixed base index but instead a type of chain index in which the weights are periodically updated (about every five years in practice).(56)

For each of the periods over which the weights are constant, the index of *OME* investment is in effect constructed by the ONS as follows:

$$QOME = w \cdot QCOMP + (1-w) \cdot QOMEXC \qquad \textbf{(52)}$$

where *QOME* is the index of total investment, set equal to 1 in the base year, *QCOMP* is a similar index for computer investment, *QOMEXC* is the index for other plant and machinery, and w is the weight for computers. This weight is the nominal share of computer investment in the total in the base year (successively 1975, 1980, 1985, 1990 and 1995). We can find the *QOME* index for (say) 1984 relative to 1985 by dividing *OME* investment in 1995 prices for 1984 by *OME* investment in 1995 prices for 1985. This works because rebasing to 1995 prices does not change growth rates for earlier periods. We can calculate the *QCOMP* index similarly. Therefore, for each period covered by a singe base, we can solve this equation for *QOMEXC*:

$$QOMEXC = [QOME - w \cdot QCOMP]/(1-w) \qquad \textbf{(53)}$$

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(56) For this reason saying that such indices are 'in 1995 prices' or 'in constant prices' is potentially misleading. It might be better to say that these series are in 'chained 1995 pounds' (copying the BEA usage of 'chained 1996 dollars').

We can then link all these fixed-base index numbers together, so that we have a type of chain index which covers the whole period. This chain index can be referenced to any year we choose, without changing its growth rate. Suppose we choose 1995 as the reference year when the index takes the value 1. Then we can multiply the chain index in each year by the nominal value of *OMEXC* in 1995, thus obtaining *OMEXC* in constant 1995 prices.

To illustrate the process, consider the following imaginary data for an *OMEXC* index calculated using equation **(53)**. Here the base periods are assumed to be periods 1 and 4 and the link period is 3.

Table D.1
Illustrative calculation of chain index from sequence of fixed-base indices

	Fixed base index		*Chain index*	
Period	*Base: Period 1*	*Base: Period 4*	*Reference: period 1*	*Reference: period 4*
1	1.000	—	1.000	0.888
2	1.050	—	1.050	0.932
3	1.070	0.950	1.070	0.950
4	—	1.00	1.126	1.000
5	—	1.10	1.239	1.100

When the reference period for the chain index is period 4, the value of the index in eg period 2 is calculated as (1.05 ÷ 1.07) x 0.95 = 0.932.

D.3 Non-additivity

In general, chain indices are non-additive: the components do not necessarily sum to the total. In other words, if we add OMEXC in 1995 prices to COMP in 1995 prices, the result will not be equal to OME in 1995 prices, except for the period 1994 to the present when the ONS has used 1995 as the base. If the component (computers) which is growing more rapidly has a falling relative price, as is the case here, then the ONS's chain index of OME grows more rapidly than the sum of the components before the base year, here 1995. This implies that the level of the ONS's chain index for OME is less than the sum of the components in constant prices in all years prior to 1994:

$$OMEXC + COMP > OME$$

or

$$OMEXC > OME - COMP$$

In other words, our (and implicitly the ONS's) chain-based estimate of the non-computer component will be greater than the estimate one would obtain by naively subtracting computer investment from total investment, in all years prior to 1994. Consequently, the growth rate of OMEXC will be *less* than the growth rate of the naïve (fixed-base) estimate prior to 1995 (since the levels are the same from 1994 onwards).

This is illustrated in Charts D.1 and D.2. The level of the naïve, fixed-base index is 25% below that of the chain index of OMEXC in 1976. Between 1976 and 1994 the fixed-base index grew at 2.34% per annum, while the chain index grew at only 0.76% per annum. Putting it another way, the sum of computer investment (COMP) and the new chain series of the total excluding computers (OMEXC) exceeds the actual total of OME investment by a growing amount as we go back further in time. By 1976 the sum of the two components exceeds the total by 33%. But to reiterate, this is just a consequence of chain-linking in the form used up to now by the ONS. That the difference between the two types of estimate is so large reflects the substantial fall in the relative price of computers which occurred over this period. If we had used the more rapidly falling US price index for computers, instead of the UK one, the difference would have been even more striking. But our aim here is to construct the series for non-computer investment which the ONS would have arrived at themselves had they chosen to do so, so we employ their methods and data.

Chart D.1

Comparison of chain and fixed base indices of *OMEXC:* levels

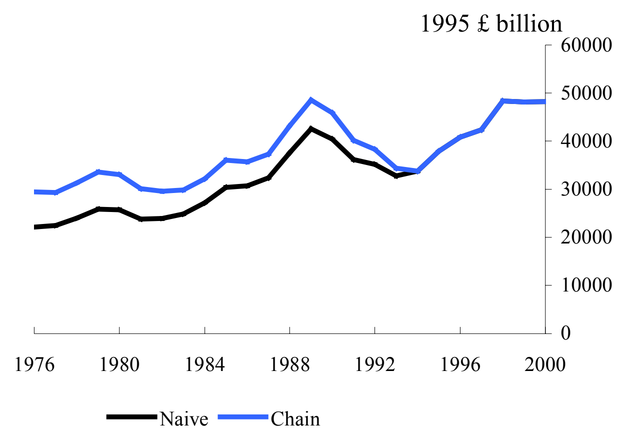
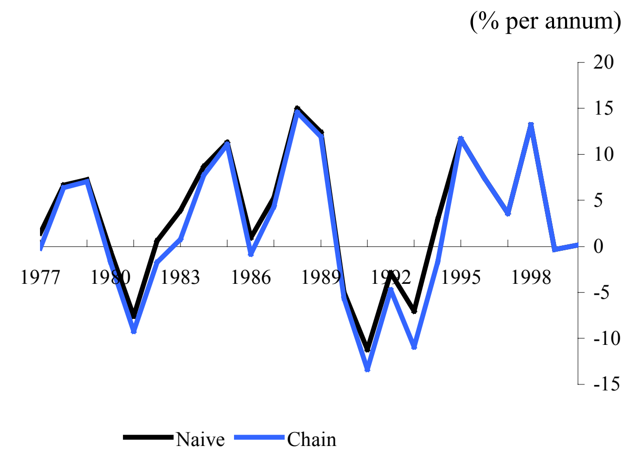


Chart D.2

Comparison of chain and fixed base indices of *OMEXC:* growth rates



Appendix E: Shares in wealth and profits and average growth rates of stocks, 1995 Q1-1999 Q4

Table E.1: Shares in nominal wealth (W) and profits (P) by asset: 1995 Q1-1999 Q4

(per cent)

Variant	Buildings		Plant		Vehicles		Intangibles		Computers		Software	
	W	P	W	P	W	P	W	P	W	P	W	P
BEA	70	42	24	45	4	10	2	3	--		--	
ONS1	65	35	31	51	3	10	1	3	--		--	
ICT1	69	43	24	38	4	10	1	3	1	5	--	
ICT2	69	43	24	38	4	10	1	3	1	6	--	
ICT3	68	40	24	36	4	10	2	3	1	6	2	6

Table E.2: Average growth rates of asset stocks: 1995 Q1-1999 Q4 (per cent per quarter)

Variant	Buildings	Plant	Vehicles	Intangibles	Computers	Software
BEA	0.68	1.52	0.81	0.36	--	--
ONS1	0.72	1.29	0.70	0.36	--	--
ICT1	0.68	0.52	0.81	0.36	6.66	--
ICT2	0.68	0.52	0.81	0.36	8.94	--
ICT3	0.68	0.47	0.81	-0.04	8.94	4.94

Table E.3: Variance of Shares in nominal wealth (W) and profits (P) by asset: 1979 Q1-2002 Q2

(per cent squared)

Variant	Buildings		Plant		Vehicles		Intangibles		Computers		Software	
	W	P	W	P	W	P	W	P	W	P	W	P
BEA	4.2	106.2	3.2	76.8	0.1	5.2	0.08	0.5	--		--	
ONS1	6.2	258.5	5.4	212.8	0.1	5.7	0.04	0.4	--		--	
ICT1	4.9	120.7	3.9	86.4	0.1	5.2	0.08	0.5	0.1	2.8	--	
ICT2	5.0	123.8	3.9	85.0	0.1	5.1	0.08	0.5	0.1	2.8	--	
ICT3	5.6	129.4	3.6	81.6	0.1	5.2	0.09	0.3	0.1	2.6	0.4	3.5

Table E.4 Variance of growth rates of asset stocks: 1979 Q1-2000 Q2 (per cent squared)

Variant	Buildings	Plant	Vehicles	Intangibles	Computers	Software
BEA	0.02	0.36	1.14	0.98	--	--
ONS1	0.01	0.17	0.77	0.98	--	--
ICT1	0.02	0.27	1.14	0.98	2.40	--
ICT2	0.02	0.27	1.14	0.98	6.98	--
ICT3	0.02	0.27	1.14	1.06	6.98	7.14

Chart E.1

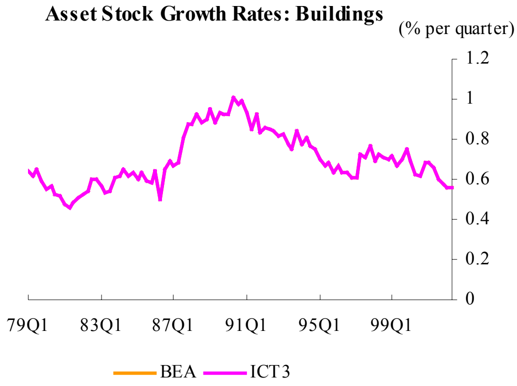


Chart E.2

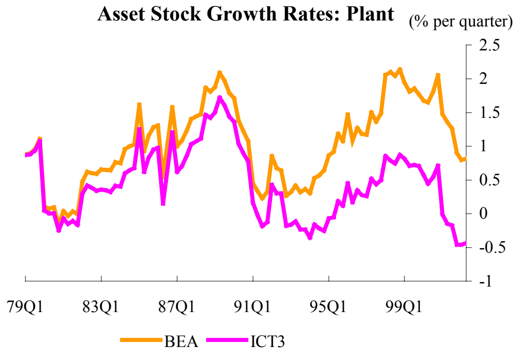


Chart E.3

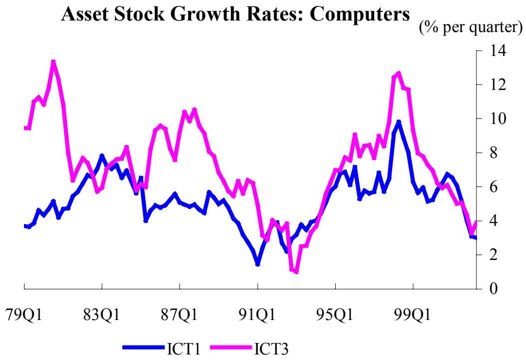


Chart E.4

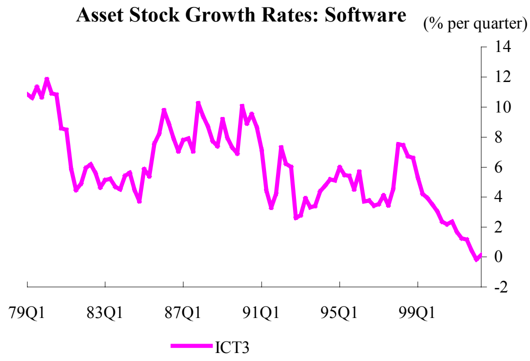


Chart E.5

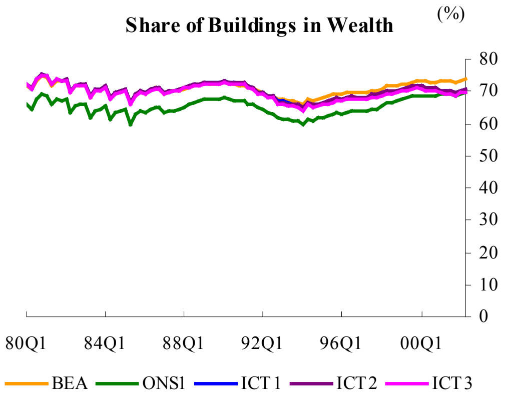


Chart E.6

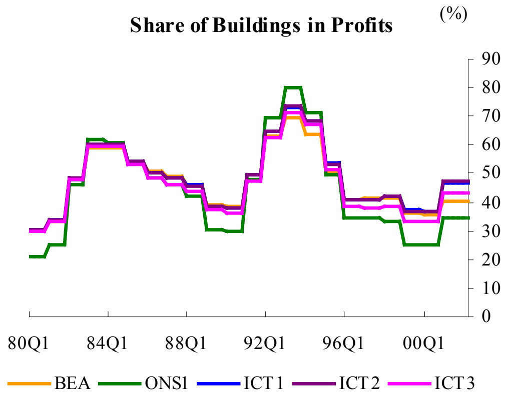


Chart E.7

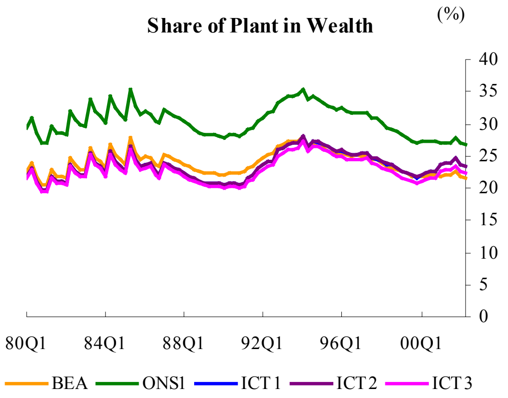


Chart E.8

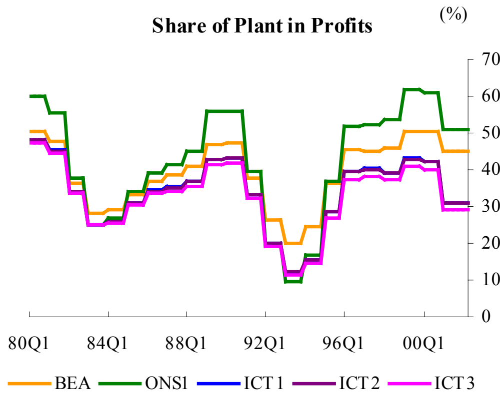


Chart E.9

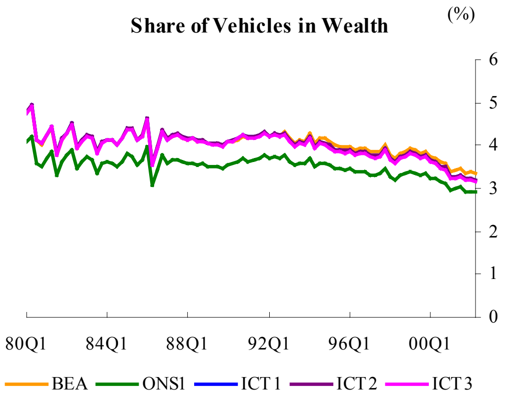


Chart E.10

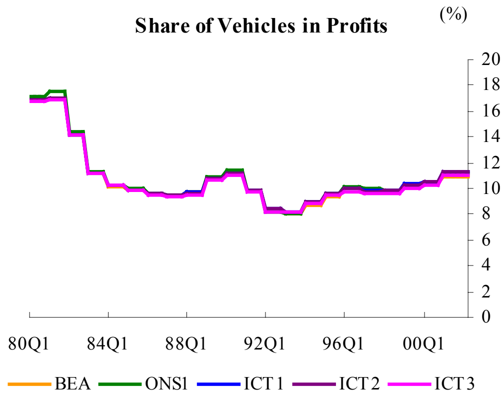


Chart E.11

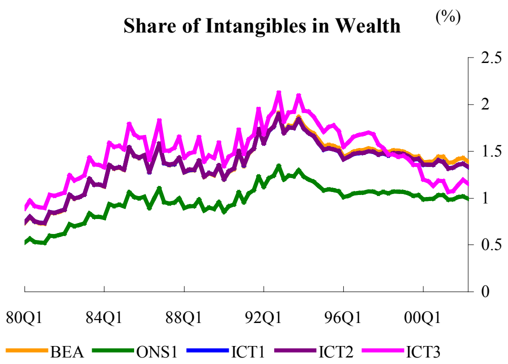


Chart E.12

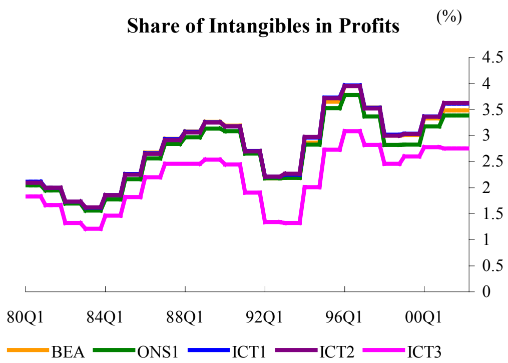


Chart E.13

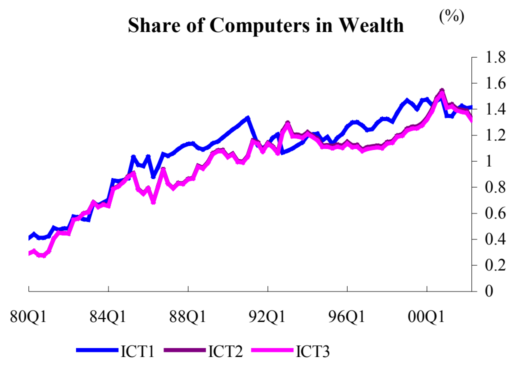


Chart E.14

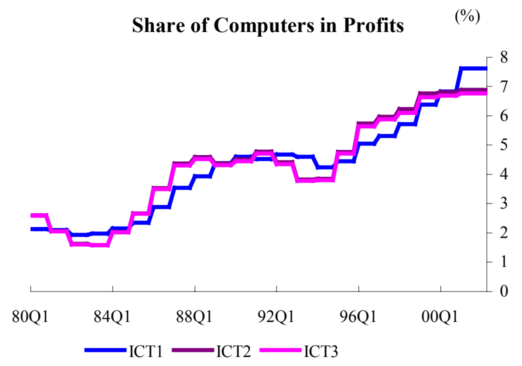


Chart E.15

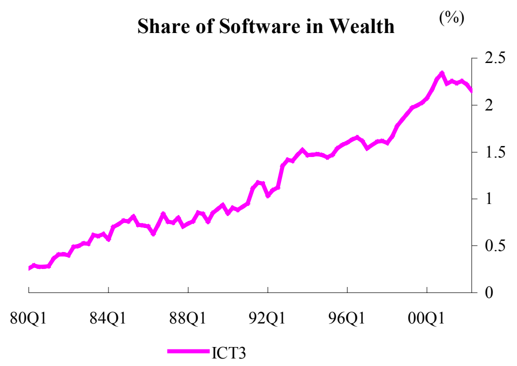


Chart E.16