Default probabilities and expected recovery: an analysis of emerging market sovereign bonds

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Abstract

We develop a simple bond pricing model to map the prices of individual EME sovereign bonds into term structures of implied (risk-neutral) default probabilities and expected recovery rates. Simple indices of bond spreads are found to be closely correlated with long-term risk neutral default probabilities, so may provide a straightforward way of monitoring shifts in investors' perceptions. But short-term risk neutral default probabilities behave quite differently, implying that there are periods of market-wide changes in volatility that do not show in measures of average spreads. Estimation of time-varying recovery rates appears to work best for countries in crisis, and suggests that expected recovery falls as the prospect of default becomes imminent. Movements in the median time to default generally appear plausible, both across time and across countries.

Summary

In this paper information contained in bond prices is backed out to assess credit risk in emerging market economies (EMEs). As a first step a model is set out which is used to decompose bond prices into its constituent parts – in particular default probabilities and expected recovery rates. The model is then applied to a group of EME sovereign bonds. This enables a judgement to be made among other things, on whether the model is useful to gain some insight into recent emerging market crises.

Yield spreads on EME sovereign bonds reflect, in part, market perceptions of the risk of default and expected recovery in the event of default. Typically, indices of average bond yield spreads are used to evaluate how the market's perception of credit risk evolves over time. However, backing out 'fundamental' determinants such as default probabilities and recovery rates is not straightforward. Moreover, there is information in the term structure on the probabilities of default in the near term that cannot be inferred from simple indices of average spreads.

There are a number of ways to extract this information but two types of models that are commonly used are structural and reduced-form (intensity-based) ones. A simple 'reduced-form' approach is followed in this paper. The model is augmented to incorporate information from the yield curve by introducing a more realistic distributional assumption for the risk-neutral probability density function. A Weibull distribution is assumed which allows the level and the slope of the probability of default structure to be derived. It also enables useful summary statistics (such as the median time to default) to be calculated which gives a greater insight into the development of credit perceptions. The model also allows time-varying recovery rates to be estimated simultaneously with the probability of default.

The model is applied to six EMEs: Argentina, Brazil, Colombia, Mexico, Russia and Turkey over the January 2000-July 2002 period. For all countries, investors' perception of the (risk-neutral) probabilities of default at different maturities and the expected half-life to default are backed out. Long-term probabilities of default are found to be highly correlated with the spread. However, short-term probabilities behave quite differently indicating that there are periods of high volatilities that seem to coincide with market-wide uncertainty. Time-varying recovery rates are assumed for countries facing financial difficulties in the short term – such as Argentina and Brazil

– and the empirical results are consistent with this assumption. In other words, investors seem to perceive that recovery rates fall significantly when default seems imminent. Finally, movements in the median time to default generally appear plausible – falling when credit conditions deteriorate and rising when they improve – both across time and country.

Notwithstanding problems with the paucity of data for some EMEs, the findings of this paper shed light on recent sovereign crises.

1 Introduction

Yield spreads on emerging market economy (EME) sovereign bonds reflect market perceptions of the risk of default and expected recovery in the event of default. Indices of average spreads on subsets of EME bonds - for example JP Morgan Chase & Co's Emerging Market Bond Index Global (EMBIG) - are commonly used as indicators of EME external financing conditions.(1) But the read-across from average spreads to 'fundamental' determinants such as default probabilities, risk premia and recovery is not straightforward. Moreover, there is information in the term structure of EME sovereign debt that cannot be inferred from indices of average spreads. In particular during times of increased credit concern, the slope of the yield curve can capture investors' perceptions of near-term default prospects. This paper develops a formal bond pricing model to map the prices of individual EME sovereign bonds into term structures of implied (risk-neutral) default probabilities and expected recovery rates.

Our bond pricing model extends the work of Merrick (2001), by utilising a more plausible distributional assumption for the risk-neutral probability density function of default. Specifically, we assume a Weibull distribution which allows us to characterise the (risk-neutral) default probability distribution by specifying two key parameters - the level and the slope of the probability of default structure. A third parameter allows recovery rates to be estimated simultaneously. Although the model we develop is highly parsimonious, we encounter some difficulties in obtaining plausible estimates of the model parameters for some countries. Coupled with the paucity of bonds of lengthier maturities in many EMEs, this suggests that bond pricing models may not be especially reliable surveillance tools. Nevertheless, the findings of this paper do shed some light on some sovereign crises in the recent past.

Simple indices of bond spreads are found to be closely correlated with long-term risk-neutral default probabilities, so may provide a straightforward way of monitoring shifts in investors perceptions. But short-term risk-neutral default probabilities behave quite differently, implying that there are periods of market-wide changes in volatility that are not apparent in measures of average spreads. Estimation of time-varying recovery rates appears to work best for countries in crisis, and suggests that expected recovery falls as the prospect of default becomes imminent. Finally, movements in the median time to default generally appear plausible, both across time and

(1) See Cunningham, Dixon and Hayes (2001) for a more detailed discussion of the Bank of England's use of data on EME sovereign yield spreads.

across countries.

The remainder of the paper is structured as follows. Section 2 describes the relevant literature. In Section 3 we set out the theoretical pricing model. In Section 4 we describe the data and estimation method. Section 5 presents our empirical findings and Section 6 concludes.

2 Relevant literature

The existing literature on pricing EME sovereign debt is quite small.

Duffie, Pedersen and Singleton (2003) (hereafter DPS) present a rich and detailed analysis of US dollar-denominated Russian bond prices, around the time of the 1998 default. They combine an extension of Duffie and Singleton's (1999) default-risky bond pricing model with factor models for each individual bond spread and the risk-free interest rate. The estimated parameters are used to make inferences about the determinants of spreads, differences in spreads across bonds, differences in expected recovery across bonds, and the correlations of spreads with factors such as official foreign exchange reserves and the price of oil.

The DPS approach, however, is not well suited to our purpose for several reasons. First, we are more interested in studying term structure effects both across time and across countries, rather than comparing the relative pricing features of different bonds of the same issuer. Second, because DPS's analysis is based on time-series models, when a new bond is issued one needs to wait until sufficient time-series observations are available to estimate the parameters of that bond's factor model before including it in the analysis. This is a concern if we want to incorporate the most up-to-date information possible. Third, although one of the major benefits claimed for the Duffie-Singleton model is that existing technology for modelling riskless term structure dynamics can be applied to default-risky assets, the appropriateness of such techniques is unclear. Yield spreads on default-risky bonds contain significant risk premia, which may behave in a similar way to those found in equities.(2) But these risk premia are notoriously difficult to model, and we are sceptical of the likely stability of the parameters of a parsimonious factor model such as that employed by DPS.

(2) For example, Elton, Gruber, Agrawal and Mann (2001) find that Fama and French's (1993) three-factor model for equities explains a significant portion of the variation in US corporate bond spreads.

Merrick (2001) devises a simple model (see below) to trace the movements of implied risk-neutral default probabilities and recovery rates for Russian and Argentinian US dollar eurobonds around the time of the 1998 Russia crisis. He finds, *inter alia*, that implied recovery rates on Russian eurobonds fell sharply after the announced default on Russian GKOs, and that the coincident sharp fall in Argentinian bond prices was driven largely by a rise in the risk-neutral probability of default. Importantly, the model is less data intensive and can thus be used to analyse EMEs with fewer bonds. Second, by focusing on the cross-section of bonds at each point in time, new information contained in the prices of new bonds can be used as soon as prices are available. It is therefore able to adapt to changes to the extant bond population relatively seamlessly across multiple countries.

3 The model

3.1 The basic model

Merrick (2001) follows other reduced-form pricing models in treating the probability of default as an exogenously determined input of a standard calculation of expected present value.(3) The default probability is combined with the contracted payment profile on the bond, and, together with assumptions about recovery in the event of default, an expected payment profile is determined. After discounting at appropriate rates, the bond price emerges.

The basic layout of the model is as follows. Let V_t^N denote the market value at time t of a default-risky zero-coupon bond with N periods to maturity.(4) We define t as being the contracted payment dates. Let h_{t+i} denote the risk-neutral probability that the issuer defaults during the period $t+i$ to $t+i+1$ conditional on having not defaulted prior to $t+i$. In the event of default, the bondholder receives a payout φ_s. The price of this bond at time t in the event of no prior default would be the present value of the probability-weighted expected payouts in the default and

(3) Models on pricing default-risky debt generally fall into one of two categories, 'structural' or 'reduced-form'. In structural approaches, variation over time in the value of the bond issuer's assets and liabilities is modelled directly, and default occurs if the asset value falls below some proportion of liabilities. The probability of default therefore arises endogenously, and depends *inter alia* on the initial level of the asset-liability ratio relative to the default trigger, and the volatility of the processes. The bondholders' loss given default is also endogenous, depending on the extent to which the asset value after liquidation falls short of the face value of outstanding bonds. This strand of literature was founded by the contingent claims approach of Merton (1974). Reduced-form models in contrast obtain the value of a bond from a standard calculation of expected present value of the bond's contracted payment profile, where both the default probability and the recovery rate are exogenous. See for example, Jarrow, Lando and Turnbull (1997) and Duffie and Singleton (1999).

(4) The extension to coupon-paying bonds is straightforward, and is presented below.

no-default states:

$$V_t^N = e^{-r_t}[(1-h_t)\, E_t^* V_{t+1}^{N-1} + h_t E_t^* \varphi_{t+1}] \qquad \textbf{(1)}$$

where r_t is the risk-free interest rate and E_t^* denotes the expectation under a risk-neutral measure (Harrison and Kreps (1979)).

In order to obtain a tractable pricing equation, we need to impose a number of assumptions on equation **(1)**, the principle one being the assumed payout in the event of default. We adopt the 'recovery of face value' (RFV) assumption, which specifies that in the event of default the bondholder loses a fixed fraction, L, of the face value of the bond, X.

Under the RFV assumption, equation **(1)** can then be solved forwards recursively to give the following present-value formula:

$$V_t^N = Xe^{-r_{t+N}}\,[1 - F\,(N)] + (1-L)\,X \sum_{i=1}^{N} e^{-r_{t+i}}\,[F\,(i) - F\,(i-1)] \qquad \textbf{(2)}$$

where $F(i)$ denotes the cumulative risk-neutral density function for default (so that $F\,(i) - F\,(i-1)$ is the unconditional probability that default occurs between time $i-1$ and time i).

3.2 The Weibull distribution

Our point of departure from Merrick's analysis is to assume that the *risk-neutral probability distribution over default can be well approximated with a Weibull distribution*. In contrast, Merrick assumes that the 'spot' risk-neutral probability of default takes a linear functional form, ie $1 - \sqrt[i]{1 - F\,(i)} = \alpha + \beta\,(t+i)$. We find this specification somewhat odd and arbitrary. In particular, as it stands there is no guarantee that forward default probabilities will be positive or less than one. We prefer to adopt the Weibull distribution, a standard tool of duration analysis (see, for example, Cox and Oakes (1984)). This is for several reasons. Owing to data limitations, we are constrained to using a model which is as parsimonious as possible. With only two parameters, the Weibull is an ideal choice. Yet, despite this parsimoniousness, the Weibull still enables us to determine the slope of the term structure, which is necessary to form a view about investors' perceptions of near-term default. Additionally, useful summary statistics, such as the median time to default for example, can easily be calculated.[5]

(5) However, the fact that this distribution does not allow for survival as time tends to infinity may mean it is not well suited to the analysis of highly creditworthy sovereigns.

Since the Weibull distribution is not commonly used in the bond pricing literature, it is important to have some understanding of how it behaves. The probability density function is

$$p_X(x) = \frac{c}{\alpha}\left(\frac{x}{\alpha}\right)^{c-1} e^{-\left(\frac{x}{\alpha}\right)^c}$$

with cumulative density function

$$F_X(x) = 1 - e^{-\left(\frac{x}{\alpha}\right)^c} \tag{3}$$

This function is described by just two (strictly positive) parameters: α and c. The parameter α affects the *level* of the default probability term structure: the smaller it is, the higher the probability of default at any time. The other parameter (c) determines the *slope* of the forward term structure. As it decreases, the probability of a near-term default increases. When $c < 1$ the gradient of the probability density function becomes monotonically downward sloping, that is the probability of a default today is greater than a default in the future. While when $c > 1$ a default in the near term is less likely than in the future.

The main practical benefit from using this distribution is that parameters describing the distribution can be used to calculate easily interpretable summary statistics. For example, the hazard function - which gives the instantaneous probability of defaulting at time x given that default has not occurred before that point - is of a particularly simple form:

$$h_X(x) \equiv \frac{p_X(x)}{1 - F_X(x)} = \frac{c}{\alpha}\left(\frac{x}{\alpha}\right)^{c-1} \tag{4}$$

The *median* time to default - or the half-life of a borrower - gives us the time horizon over which there is a 50% probability that the borrower will default. This statistic is also easy to compute and can be expressed as:

$$\alpha(\log 2)^{1/c}$$

Alternatively, we can compare 'spot' default probabilities across countries at fixed time horizons. We take the annualised risk-neutral probability of default at any point during the next three years, $1 - \sqrt[3]{1 - e^{-(3/\alpha)^c}}$, as a measure of near-term default-risk pricing, whereas the annualised default probability over ten years provides a metric of longer-term risk pricing.

3.3 Other assumptions

In implementing our model, we make two further important assumptions.

We assume that *the risk-free rate and the risk-neutral probability density for default are independent*. This assumption is made, *inter alia*, by Merrick (2001) in the sovereign context, and Jarrow, Lando and Turnbull (1997) in their study of corporate bonds. Ideally we would not want to impose this assumption. Countries that have large amounts of short-term external debt (and therefore require a high rate of debt rollover) and floating-rate debt are more likely to run into liquidity problems when 'risk-free' interest rates rise, and it seems likely that any liquidity crisis contains a significant risk that the country will be forced into default. However, estimation of the joint distribution of these variables would greatly complicate the analysis and expand the number of parameters that would need to be estimated. Imposing the independence and Weibull assumptions on equation **(2)** leads to the following zero-coupon bond pricing equation:

$$V_t^N = Xe^{-r_{t+N-1}}e^{-\left(\frac{N}{\alpha}\right)^c} + (1-L)\,X\sum_{i=1}^{N} e^{-r_{t+i-1}}\left[e^{-\left(\frac{i-1}{\alpha}\right)^c} - e^{-\left(\frac{i}{\alpha}\right)^c}\right] \qquad \textbf{(5)}$$

Our last key assumption regards the timing of the recovery payment in the event of default. In the sovereign context (but also in the corporate context), a great deal of uncertainty typically surrounds the resolution of negotiations to write-down debt, so any assumption on the timing of the recovery payment is to some degree arbitrary. We follow Merrick in assuming that *default can occur only on contracted payment dates* - in the case of zero-coupon bonds, this means on the redemption date of one of the bonds - and default on the payment of any bond triggers immediate default on all other bonds of the same issuer. The recovery payment is assumed to accrue immediately on all bonds.

As discussed above, because an 'insolvent' sovereign remains a going concern, once-and-for-all payments are rarely actually made. Rather, following an announced or *de facto* default by a sovereign, the prices of all bonds of equal seniority issued by that sovereign (or at least those that include cross-default clauses) will move to the expected value of the instrument with which they will likely be replaced. It is the latter value that the recovery payment in this model attempts to capture.

The model can be extended to coupon-paying bonds by assuming that the net present value of the coupons in the case of no-default is equivalent to that of a portfolio of zero-coupon bonds whose maturities extend from the first coupon date to the principal repayment date. In the event of default, we continue to assume recovery of face value. Default can occur on any contracted repayment date (coupon or principal), but only the principal payment is restructured and coupon payments are written off in full.[6] The resultant pricing equation is then:

$$V_t^N = Xe^{-r_{t+N-1}}e^{-\left(\frac{N}{\alpha}\right)^c} + \sum_{i=1}^{N} C_i e^{-r_{t+i-1}} e^{-\left(\frac{i}{\alpha}\right)^c} + (1-L)\,X \sum_{i=1}^{N} e^{-r_{t+i-1}} \left[e^{-\left(\frac{i-1}{\alpha}\right)^c} - e^{-\left(\frac{i}{\alpha}\right)^c} \right] \quad \textbf{(6)}$$

In principle this model can be estimated to determine α, c and L jointly (the parameters of the default probability density function and the loss rate). However, we can also simplify and fix L exogenously. This is appropriate if we believe that investors have 'outside information', for example they have good historic recovery rate information. This could also be necessary to add a degree of freedom if we were facing data limitations. As discussed below, this latter concern is a very relevant issue for our sample of countries.

4 Data and estimation

Our primary interest is in describing movements in US dollar bond prices on a daily basis for individual EMEs. The choice of bonds to include reflects a trade-off between including as many bonds as possible for each country, and ensuring that their prices are an up-to-date assessment of the market's assessment of credit risk. We restrict our choice to plain vanilla (fixed coupon, bullet repayment) US dollar-denominated sovereign bonds in order to develop a transparent pricing model that is potentially applicable to as wide a set of countries as possible.[7]

We take as our starting point bonds that meet JP Morgan Chase and Co's liquidity criteria for inclusion in the Emerging Markets Bond Index Global (EMBIG),[8] namely those instruments with a minimum face value outstanding of US$500 million, and whose daily prices can be verified by JP Morgan Chase & Co. We add to these bonds that have less than twelve months to maturity (such bonds are excluded from the EMBIG) and a small number of other bonds for which we are able to obtain what appear to be reliable prices. Newly issued bonds are included as soon as

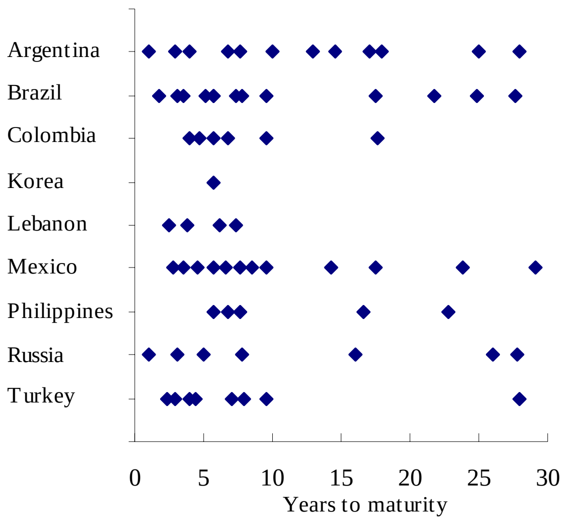
(6) When a country is near or in crisis, it is common to see discussions of the write-down in the *face* value of the country's debt that would be needed to ensure future sustainability.

(7) Our preferred instruments are liquid plain vanilla eurobonds, for two reasons. First, the complexity of addressing country and instrument-specific idiosyncracies in contracted payment and collateral structures militates against our goal of developing a simple model that can be applied easily across countries and over time. Second, the eurobond market has in recent years overtaken the Brady bond market in terms of trading volume, and the general trend is for countries to retire Brady bonds and replace them with larger, more liquid, eurobonds. Looking ahead, therefore, our model is likely to enjoy wider applicability. On the flipside, however, our focus on liquid eurobonds constrains both the time period and the number of countries for which an analysis is feasible. It also means that parametric parsimoniousness is at a premium.

(8) The EMBIG also includes Brady bonds, traded loans and (until May 2002) local market debt instruments issued by sovereign and quasi-sovereign entities. See JP Morgan (1999) for more details.

prices are available, but bonds trading on a 'when-issued' basis are excluded. Daily closing prices are taken from Bloomberg.

Chart 1: Residual maturity of sovereign eurobonds for selected emerging markets, end-June 2002



Source: JP Morgan Chase & Co.

The number of countries for which this type of analysis is feasible is limited. As Chart 1 illustrates, the pool of outstanding US dollar-denominated bonds varies widely across individual emerging markets. Some countries, such as Brazil and Mexico, have bonds distributed throughout the maturity spectrum up to around 30 years. Others have a fairly limited maturity structure - for example, Lebanon's bonds have a maximum maturity of seven years. Still more countries, for example Korea, have only one or two actively traded bonds, making term structure analysis impossible. Moreover, the development of a liquid eurobond market is a relatively recent phenomenon - analysis for most of the countries in our sample only becomes feasible in 1999 or 2000.[9] These data issues, in particular the lack of long-term bonds, constrain our analysis severely. It will take a substantial deepening in capital markets before this type of estimation can be extended to a wider array of EMEs.

We restrict our analysis to countries with at least five eligible bonds outstanding. Six countries

(9) In 2003, several major EME sovereigns (including Mexico, Venezuela, Poland and Brazil) have continued to replace some (or all) of their Brady bonds debt stock for eurobonds, thus increasing the stock and enhancing liquidity.

Table A: Eurobonds used in model estimation

Argentina	Brazil	Colombia	Mexico	Russia	Turkey
$9\frac{1}{4}$% 2001[2]	$11\frac{5}{8}$% 2004	$10\frac{7}{8}$% 2004	$9\frac{3}{4}$% 2001[2]	$9\frac{1}{4}$% 2001[2]	$11\frac{7}{8}$% 2004
$8\frac{3}{8}$% 2003	$9\frac{5}{8}$% 2005[1]	$10\frac{1}{2}$% 2006[1]	$9\frac{3}{4}$% 2005	$11\frac{3}{4}$% 2003	$9\frac{7}{8}$% 2005[4]
11% 2005	$10\frac{1}{4}$% 2006[1]	$7\frac{5}{8}$% 2007	$8\frac{1}{2}$% 2006[1]	$8\frac{3}{4}$% 2005	$11\frac{3}{8}$% 2006[1]
11% 2006	$11\frac{1}{4}$% 2007[1]	$8\frac{5}{8}$% 2008	$9\frac{7}{8}$% 2007	10% 2007	10% 2007[4]
$11\frac{3}{4}$% 2009	$11\frac{1}{2}$% 2008[1]	$9\frac{3}{4}$% 2009	$8\frac{5}{8}$% 2008	11% 2018	$12\frac{3}{8}$% 2009
$11\frac{3}{8}$% 2010[1]	$9\frac{3}{8}$% 2008	$11\frac{3}{4}$% 2020[1]	$8\frac{3}{8}$% 2009		$11\frac{3}{4}$% 2010[1]
$12\frac{3}{8}$% 2012[1]	$14\frac{1}{2}$% 2009		$9\frac{7}{8}$% 2010[1]		$11\frac{1}{2}$% 2012[1]
$11\frac{3}{4}$% 2015[1]	12% 2010[1]		$8\frac{3}{8}$% 2011[1]		$11\frac{7}{8}$% 2030[1]
$11\frac{3}{8}$% 2017	11% 2012[1]		$7\frac{1}{2}$% 2012[1]		
$12\frac{1}{8}$% 2019[3]	$12\frac{3}{4}$% 2020[1]		$11\frac{3}{8}$% 2016		
12% 2020[1,3]	$8\frac{7}{8}$% 2024[1]		$8\frac{1}{8}$% 2019[1]		
$9\frac{3}{4}$% 2027	$10\frac{1}{8}$% 2027		$11\frac{1}{2}$% 2026[1]		
$10\frac{1}{4}$% 2030[1,3]	$12\frac{1}{4}$% 2030		8.3% 2031[1]		

[1] Issued during the monitoring period.

[2] Matured during the monitoring period.

[3] Removed from the EMBIG at end-June 2001, following the June mega-swap.

[4] Removed from the EMBIG during 2001, due to lack of liquidity.

meet this requirement for some or all of the period from January 2000 to end-June 2002: Argentina, Brazil, Colombia, Mexico, Russia and Turkey. For two countries, Argentina and Russia, we do not present parameter estimates after November 2001. In the case of Argentina, this is because in the run-up to the domestic debt restructuring and default in December 2001, there appears to have been a breakdown between the bond prices and the contracted cash flows on the bonds, presumably reflecting an expected default/exchange. For Russia, the redemption of the $9\frac{1}{4}$% 2001 meant that it had only four remaining eligible bonds. Table A lists the bonds used in our estimation, and indicates those that were issued during the estimation window.

The use of risk-neutral default probabilities in our model means that we can discount future cash flows at the risk-free rate. To do that, we need a risk-free discount factor for every possible cash-flow date and, hence, require a continuous model for the term structure of risk-free interest rates. Because our bonds are all US dollar denominated, we use yields on US Treasury bonds as our proxy for risk-free rates. The US Treasury yield curve is fitted using a Svensson (1994) parametric formulation. This choice reflects the ready availability of Svensson parameter estimates for the US Treasury curve (the Bank of England calculates these on a daily basis) and the relative ease of calculating spot rates at any horizon. It is important to note however, that since we use *risk-neutral* default probabilities the results can be harder to interpret. In particular, we cannot separate actual default probabilities from risk premia. When default probabilities change, we do not know whether it is the probabilities or the risk premia that are changing.

The parameters are estimated by minimising the (equally weighted) sum of the squared pricing errors across the bonds in the sample. The details of the estimation technique can be found in Merrick (2001).[(10)] In general, convergence is obtained rapidly in particular for 'crises countries'. However, for countries with fewer bonds or with low probabilities of default, the model sometimes had trouble converging (see below). Nevertheless, convergence is generally robust to starting values, so for ease of replicability we seed the optimisation at each observation date with a standard set of initial parameter estimates.[(11)]

5 Empirical results

Our model specification enables us to:

- Estimate solely the implied default probabilities by fixing the recovery rate.
- Estimate the implied recovery rate and the default probabilities jointly.

Data limitations lead us to fix the recovery rate for most of our countries. This increases degrees of freedom and improves the robustness of our results, in particular for countries with fewer bonds. Furthermore, we find that, for creditworthy countries, the model has difficulty differentiating between the level parameter (α) and the recovery rate.[(12)] It thus seems sensible to assume constant recovery rates. It is conceivable that for most countries in our sample, over our two-year observation period, expectations of recovery have not changed substantially. We fix it at the same level for these countries: 40%, broadly in line with average historical experience of recovery on defaulted corporate bonds and with the write-downs incurred during recent sovereign bond restructurings (Izvorski (1997)).

(10) See Merrick (2001) Section 4, pages 1,929-30.

(11) The initial values are: $\alpha = 20$, $c = 1$, $recovery = (1 - L) = 50\%$. In some cases, sensible parameters were not obtained using these values. In these cases, setting the starting values equal to the previous day's estimates produced more reliable results.

(12) In simulation, we find that the objective function is not well behaved for creditworthy countries, particularly when they have many short-term bonds. In these cases, the estimation becomes unstable and often leads to implausible results for both the recovery rate and the levels parameter.

However, for countries with near-term financing difficulties fixing the recovery rate might be an unrealistic assumption. As default approaches, investors could be reassessing the recovery rate at a higher frequency. We have three such ‘crises countries’ in our sample: Argentina, Brazil and Turkey. Argentina and Brazil have a large enough selection of bonds spanning different maturities to attempt this analysis. And in fact, for these countries we obtain more plausible results when estimating the recovery than when fixing it. But Turkey has few bonds of long maturities, and we fix the recovery rate in this case. It should be noted that estimating the recovery rate jointly may have implications for the estimated default probabilities. This means that it would be harder to compare them to countries for which the recovery rate is fixed.

In summary, we fix the recovery rate for all countries except Argentina and Brazil. The estimation results are presented below. We proceed to describe how default probabilities, median time to default and recovery rates are evolving for our countries. We then explain under which conditions our model works best.

5.1 Default probability dynamics

Charts 2 to 7 show the estimated three-year and ten-year implied risk-neutral default probabilities for the countries in our data set, together with each country’s EMBI Global subindex yield spread. Table B shows statistical correlation coefficients between changes in EMBIG spreads and default probabilities.[13]

From Charts 2 to 7 it seems that the EMBI Global indices are reasonably highly correlated with the long-term risk-neutral default probabilities. We see, for example, that in Argentina long-term default probabilities increased progressively in line with the spread as creditworthiness concerns mounted in 2001. The only exception is Brazil where at the end of 2001 long-term risk-neutral probabilities did not rise with the spread. Instead, the short-term probability of default rose and the recovery rate fell by half (see Section 5.3).

(13) The table shows correlation coefficients between *changes* in spreads and default probabilities. We do not calculate them on *levels* as, due to non-stationarity, the statistic can be misleading.

Table B: Correlation coefficients between changes in EMBIG spreads and estimated risk-neutral default probabilities

		Argentina	Brazil	Colombia	Mexico	Russia	Turkey
Correlation Coefficient	3-yr default prob & EMBIG spread	0.10	0.15	0.05	0.06	0.09	0.16
	10-yr default prob & EMBIG	0.03	0.32	0.53	0.53	0.23	0.43

Although there is a reasonable degree of concordance between changes in the long-term risk-neutral probabilities and the EMBI Global country subindices, the short-term risk-neutral default probabilities generally display a different pattern (for obvious reasons, given the sample period, Argentina is a notable exception). The short-horizon default probabilities are more volatile, reflecting a combination of genuinely higher volatility in near-term risk pricing and probably a degree of overfitting. One interesting feature is the pickup in volatility of the short-horizon default probabilities across all countries in the second half of 2000, as crises in Argentina and Turkey emerged. This cross-country effect seems likely to reflect genuine widespread uncertainty about short-term prospects for emerging market debt. This aspect of cross-market contagion that would not be picked up looking at average spreads.

5.2 Median time to default

Charts 8 to 10 show time series of the median time to default, or implied half-life.[14] We plot this statistic as it has several advantages over the EMBIG spread. As it is a *median* time it embodies the term structure of the distribution. Also, despite being a risk-neutral measure (making it hard to interpret the level), the trend is informative and easily comprehensible. If we see for example that this statistic is rapidly falling over time, it means that the situation is worsening.

(14) Note that because the median time to default: is based on *risk-neutral* probabilities it does not give a direct indication of the *actual* median time to default, although that will be one of its determinants.

Chart 2: Implied risk-neutral default probabilities, and EMBI Global subindex spreads - Argentina (estimated recovery)

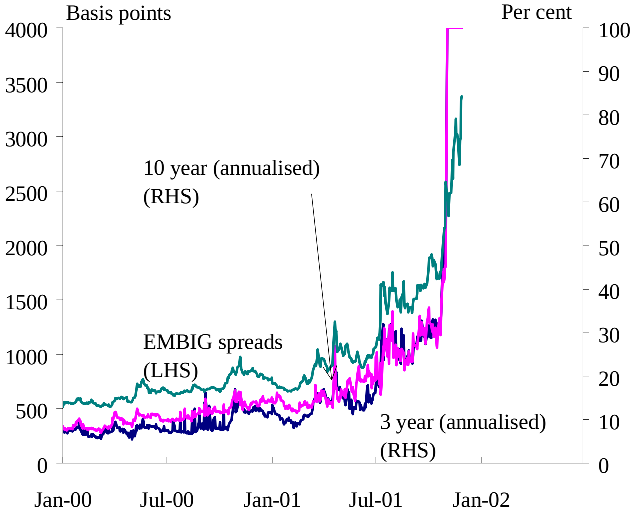


Chart 3: Implied risk-neutral default probabilities, and EMBI Global subindex spreads - Brazil (estimated recovery)

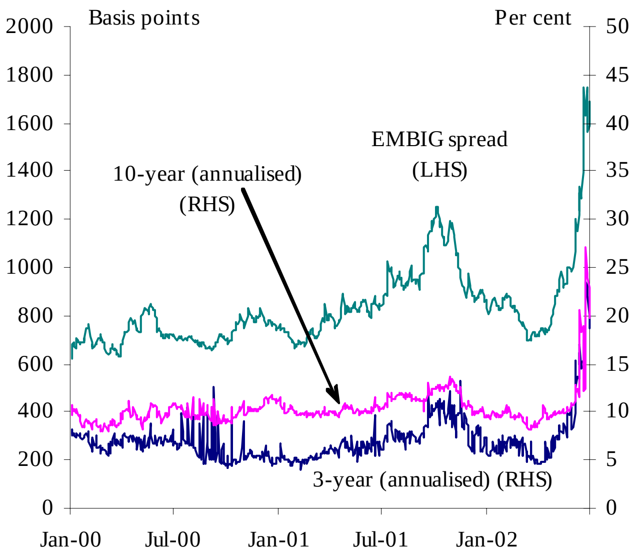


Chart 4: Implied risk-neutral default probabilities, and EMBI Global subindex spreads - Mexico (fixed recovery)

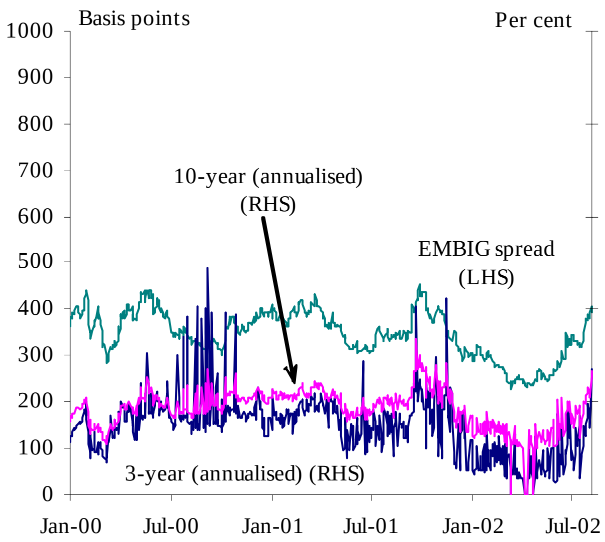


Chart 5: Implied risk-neutral default probabilities, and EMBI Global subindex spreads - Colombia (fixed recovery)

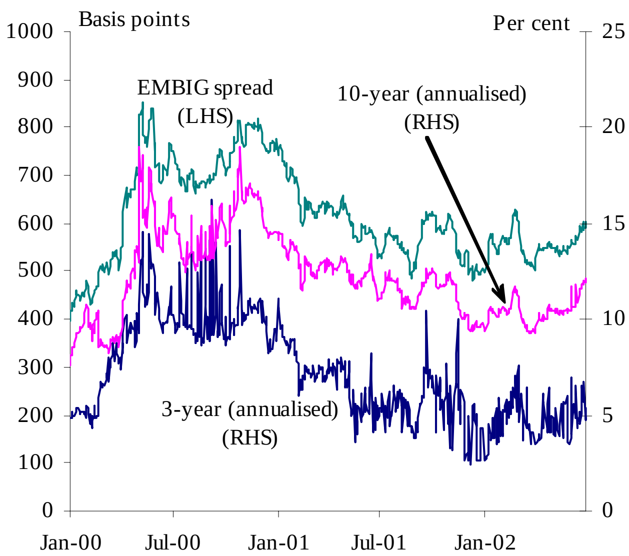


Chart 6: Implied risk-neutral default probabilities, and EMBI Global subindex spreads - Turkey (fixed recovery)

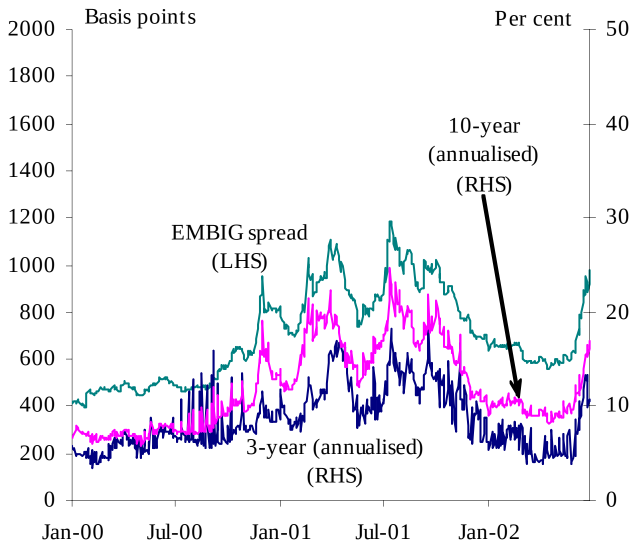


Chart 7: Implied risk-neutral default probabilities, and EMBI Global subindex spreads - Russia (fixed recovery)

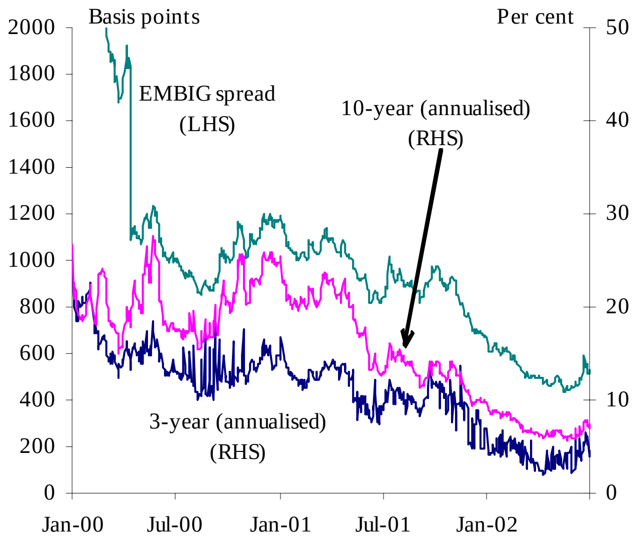


Chart 8: Implied median time to default - Argentina and Brazil (estimated recovery)

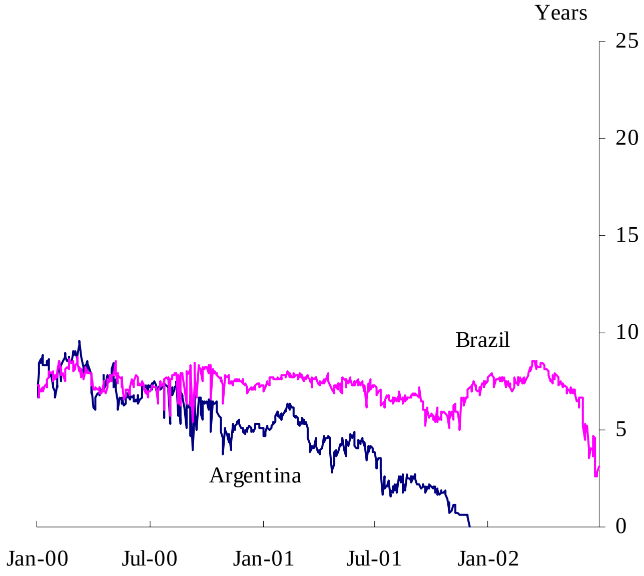


Chart 9: Implied median time to default - Colombia and Mexico (fixed recovery)

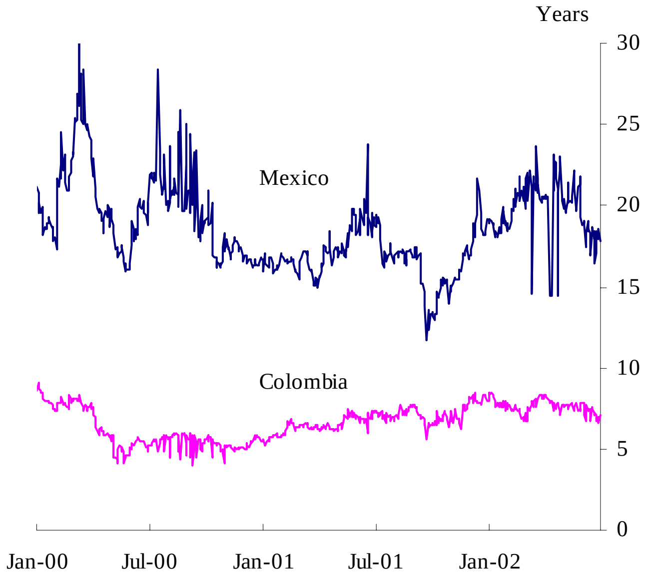
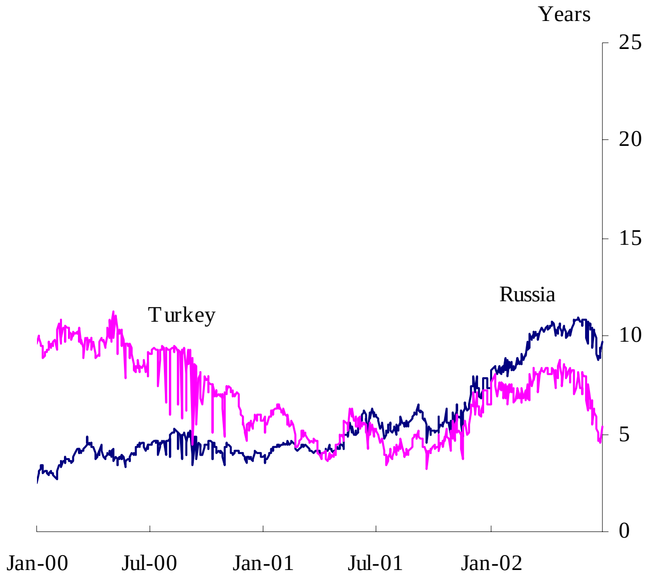


Chart 10: Implied median time to default - Russia and Turkey (fixed recovery)



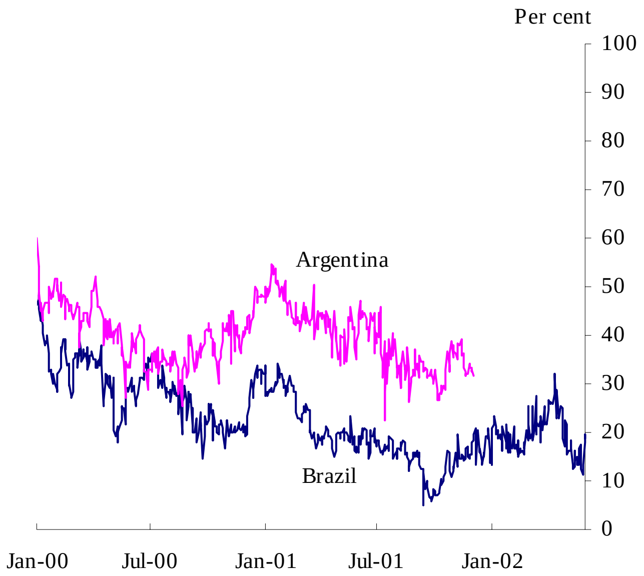
On this measure, Mexico appears to be the least vulnerable country by some way. The improvement in the outlook for Russia is evident, in particular from the end of 2001. While the steadily worsening situation in Argentina over the period is reflected in a declining half-life tending to zero as the situation becomes critical. The more recent worsening in prospects in 2002 for Brazil and Turkey also show up clearly as a marked decline in this measure. Given the plausibility of the relative levels and moments, overall, the implied half-life offers a useful summary statistic to assess developments in risk, both across time and across countries.

5.3 Recovery rates

By relaxing the assumption of an invariant recovery rate we can observe its evolution over the sample period. We achieve some informative results for sovereigns with substantial near-term financing needs. Chart 11 shows the estimated recovery rates for Argentina and Brazil. Although the estimated recovery rates are implausibly volatile from day to day, the longer-term picture appears plausible.

In Argentina, Chart 11 shows that recovery peaked at around 50% in January 2001, coinciding with the announcement of an IMF package which seems to have delivered a significant but temporary boost to expected recovery. During the rest of 2001, implied recovery declined steadily, to around 30% immediately prior to the announced default, comparable to that prevailing in mid-2000 and also consistent with the haircut proposed by the government in late 2004.

Chart 11: Implied recovery rate - Argentina and Brazil



In Brazil, the implied recovery rate is materially lower than in Argentina - averaging around 25%. It is, however, difficult to identify evidence which would explain why investors might be pricing in such different recovery assumptions. Although the level of the recovery rate might be questionable, the trend generally appears plausible. Brazilian recovery declined in line with that on Argentinian bonds during 2001. One reason for this might have been fear of policy contagion from Argentina. In other words, if the policies put in place by the Argentine authorities were perceived to be destroying the value of collateral and investors were worried that other countries facing payment difficulties might follow the same strategy, then they would reassess their recovery assumptions. During the first half of 2002, implied recovery increased steadily but declined sharply thereafter. This fall might reflect uncertainty about the effectiveness of government policies to preserve bondholder value under a new administration. However, in 2003 (not shown in chart), as investors became more confident about the Lula government, the recovery rate increased back to 30%.

For both countries, the correlation between the recovery rate and default probabilities is weakly negative. This could be interpreted as confirmation that when default probabilities rise investors reassess their recovery rate assumptions more frequently as they become simultaneously more pessimistic about the likelihood and size of repayment.

5.4 When does the model work best?

As discussed above, when estimating recovery rates the model works reasonably well for Argentina and Brazil. However, it works less well for the other countries. So for what kind of data does the model work best?

Unsurprisingly, the model works better as more bonds are available. When fixing the recovery rate it is found that stable results necessitate at least five bonds whereas when estimating the recovery rate, many more bonds are needed. Furthermore, to obtain plausible estimates of recovery, one needs to have a selection of bonds evenly spanning different maturities (in particular long-term maturities). This is why estimating recovery rates for countries such as Turkey (only one bond over 15 years maturity) or Russia (only five bonds) is difficult. Nevertheless, as more EME sovereigns issue tradable debt and exchange their Brady bonds for eurobonds a greater panel of countries will be available for analysis.

The more liquid the bonds the better the results. Many EME bond markets are illiquid, so that substantial mispricing can persist for a considerable time. This liquidity issue is more problematic for a country with few bonds (as each individual bond will have a greater impact on the estimation) in particular at certain maturities. Thus for example, if the longer-term bond is trading at a premium owing to its low liquidity, it will have a profound impact on the recovery rate estimation.

The further a country is from default, the more important it is that it has many liquid *long-term* bonds. The pricing function has the feature that the higher α (the lower the long term probability of default) the higher the impact long-term bonds have on determining the long-term parameter and the recovery rate. So if one wants to estimate recovery rates for more creditworthy countries it is crucial that they have many liquid long-term bonds (even ignoring some shorter-term bonds if need be). However, to get plausible results, we generally recommend fixing the recovery rate for these countries.

5.5 Robustness

We assess the robustness of the model by answering the following two questions:

1) Do the parameter estimates seem closely interrelated, even when there is no trend in the data?

The simultaneous estimation of default parameters and recovery rates in such a framework often suffers from the close interrelation of the parameters. Typical results show high correlations between estimated default parameters and recovery parameters even when market prices do not show a clear trend. Table C shows the correlation coefficients between parameter estimates. None of the correlation coefficients is very high, and for Brazil and Argentina we note that the interrelation between the recovery rates and default parameter are equally low.

Table C: Summary statistics of parameter estimates for countries in our data set

		Argentina	Brazil	Colombia	Mexico	Russia	Turkey
Average	α	8.01	9.19	8.46	18.17	7.57	9.10
	c	1.27	1.34	1.51	1.25	1.36	1.29
	RR	42.8	22.9	n.a.	n.a.	n.a.	n.a.
	MSE	0.40	0.38	0.37	0.35	0.45	0.49
Correlation Coefficient	α & c	0.23	0.49	0.43	0.02	0.29	-0.25
	α & RR	0.04	0.26	n.a.	n.a.	n.a.	n.a.
	c & RR	0.45	0.21	n.a.	n.a.	n.a.	n.a.

RR - Recovery Rate

MSE - Mean square errors

2) Does the addition or removal of bonds during the time series lead to jumps in the parameter estimates?

Table D shows the average absolute change in the parameter values when a bond enters/leaves the time series. The parameters seem relatively stable, in particular when compared to the standard deviation over the whole sample period. Unsurprisingly, when there are few bonds the addition of a new one can have a slight impact on the results but this remains marginal.

Table D: Average absolute change in parameter values when bond enters/leaves the time series and standard deviation of parameters over the sample period

		Argentina	Brazil	Colombia	Mexico	Russia	Turkey
Average absolute change	α	0.59	0.25	0.53	0.38	n.a.	0.38
	c	0.06	0.04	0.21	0.04	n.a.	0.15
	RR	2.84	2.61	n.a.	n.a.	n.a.	n.a.
Standard deviation	α	2.52	1.57	1.23	2.58	2.87	2.77
	c	0.27	0.19	0.22	0.25	0.18	0.17
	RR	8.79	7.57	n.a.	n.a.	n.a.	n.a.

Overall, these results suggest that our model is robust.

6 Conclusions

We present an extension of Merrick's (2001) model for pricing emerging market sovereign bonds. We find that the model works best for countries with a good extant stock of bonds spanning a wide range of maturities, and for countries with near-term financing difficulties. We report some plausible and interesting findings for two large EMEs: Brazil and Argentina. Implied recovery rates can provide useful insights into investor perceptions and valuations. Long-term risk-neutral default probabilities are closely correlated with indices of average spreads, suggesting that the latter may be a low-cost way of monitoring the former. Spread indices do not, however, pick up variation in short-term risk-neutral default probabilities. Although these are volatile, they do point to periods of market-wide uncertainty about the near-term prospects for emerging market debt. Finally, the median time to default, or implied half-life, appears to be a useful summary measure both of developments over time in the pricing of credit risk on a country's sovereign bonds, and of relative risk pricing across countries.

Although the implementation of our model casts light on some recent events, the relative lack of depth in EME capital markets means that only few countries can be analysed in this fashion. This limits the scope of such models for forward-looking analysis. The model estimates are also too volatile on a day-to-day basis to be of use for very high frequency analysis. But as the selection of eurobonds increases over time and as the market for emerging market sovereigns bonds becomes more liquid, it may be possible to broaden our analysis to include more countries and to generate more robust estimates.

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