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Mapping systemic risk in the international banking network

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Abstract

Systemic risk among the network of international banking groups arises when financial stress threatens to criss-cross many national boundaries and expose imperfect international co-ordination. To assess this risk, we apply an information theoretic map equation due to Martin Rosvall and Carl Bergstrom to partition banking groups from 21 countries into modules. The resulting modular structure reflects the flow of financial stress through the network, combining nodes that are most closely related in terms of the transmission of stress. The modular structure of the international banking network has changed dramatically over the past three decades. In the late 1980s four important financial centres formed one large supercluster that was highly contagious in terms of transmission of stress within its ranks, but less contagious on a global scale. Since then the most influential modules have become significantly smaller and more broadly contagious. The analysis contributes to our understanding as to why defaults in US sub-prime mortgages had such large global implications.

Key words: Networks, international banking groups, systemic risk, information theory.

JEL classification: F2, F3.

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Summary

An astonishing feature of the 2008 financial crisis was how quickly and extensively the relatively small write-downs in US sub-prime mortgages spread to a situation where only two years later governments worldwide had to provide massive support to their banking systems. International banks played a key role in transmitting contagion through their claims on each other. This paper examines how the interconnectedness of the international banking system impacts the threat of systemic risk in the international banking network.

Cross-sectional systemic risk is the potential for shocks that hit one part of the system to be transmitted to the rest of the system. This potential can be analysed in a variety of ways. However, all approaches look at connections between different entities that are reflected in their balance sheets. A straightforward approach is to simulate shocks to bank balance sheets and examine the repercussions. However, this involves making many assumptions about the type and size of shock, how widespread it is, and how banks adjust to its occurrence. Our approach abstracts from specific details about shocks and looks more at the contagious capacity of the network.

The data we use are the 420 external claims that 21 international banking groups held on each other for each quarter over nearly 25 years. This data set was compiled by the Bank for International Settlements and banking groups are defined by the country where banks do their business.

The aim is to simplify the raw data on claims and liabilities into a map that succinctly summarises how financial contagion moves between international banking groups. We begin by specifying a network of financial linkages in which banks transmit stress to each other via two channels, a funding channel and a lending channel. Stress is transmitted through the funding channel when a bank refuses to rollover a loan and it is transmitted through the lending channel when a bank defaults on a loan. We then apply a network clustering technique developed by physicist Martin Rosvall and biologist Carl Bergstom to determine the most parsimonious yet accurate description of the network that can be used to map the movements of an imaginary traveller, whom we refer to as Mr Contagion. Because this approach is based

on tracking movement, it is well suited to help draw a map for the contagion of financial stress.

Under this approach, clusters are formed when stress travels between the members of a cluster with sufficiently greater intensity than it does to the banking groups outside the cluster. As such, a cluster can be thought of as a collection of banking entities that are so interconnected that they can be treated as one group.

Clustering is done at each date from 1985 Q1 to 2009 Q3. The changes in clustering that are observed capture well-known changes in the international banking landscape that have occurred over the past quarter century. In the late 1980s, Japanese resident banks expanded their overseas operations and this move is reflected by the inclusion of Japan in a large supercluster, along with the United Kingdom, the United States and the Cayman Islands. That cluster breaks up by the beginning of the 1990s due to the emergence of the Japanese banking crisis. Over the next decade and a half, European banking groups increase in relative importance and accordingly we see many smaller, but still influential, clusters appear in our maps.

Changes in clustering only tell part of the story. We also examine the extent to which the international banking network became more broadly contagious over time. To do this it is necessary to choose a benchmark modular structure and examine changes in the extent to which contagion spreads out of the fixed clustering. The benchmark we use is the clustering for 1989 Q3 when the United Kingdom, Japan, the United States and the Cayman Islands were combined into one module. This allows us to see how the systemic risk associated with financial problems that originate within these major financial centres increased over time. The amount of contagion flowing outside the fixed modules from 1989 Q3 increased since the end of the 1980s and it peaked in 2008 Q2, just before Lehman Brothers' default, but still remains at a relatively high level.

It is important to understand that our results cannot be used to infer anything about the current riskiness of the system. The reason for this is that our contagion analysis only concerns the cross-sectional component of systemic risk and offers no insights as to changes in the average quality of banks' balance sheets over time.

1 Introduction

An astonishing feature of the 2008 financial crisis was how quickly and extensively the relatively small write-downs in US sub-prime mortgages spread to a situation where only two years later governments worldwide had to provide massive support to their banking systems. In the years prior to the crisis, large banking groups had become highly interdependent across national borders through a complex web of direct claims on each other, ownership structures and other risk transfers and also through participation in common markets.[1] Because the system was so intertwined, the financial crisis was transmitted rapidly through default chains, funding squeezes, fire-sale externalities and a rash of counterparty fear. In this paper we use network theory to help understand the transmission of stress in this complex financial system.

Our focus is on the international banking network. The international banking network is a set of bilateral claims (links) of different banking entities (nodes) on each other. Nodes are initially defined by separating banking groups (all the banks operating in a particular country) into their funding and credit arms; each node is a funding or credit arm of a particular banking group. This separation allows us to distinguish between two different channels of contagion. Banks defaulting on loans transmit stress to their creditors via a credit channel. This is a situation where a problem at one banking group's funding arm is transmitted to another banking group's credit arm. However, it was also observed during the crisis that banks got in trouble because their creditors refused to keep lending to them — a funding channel. This is a situation where stress flows from the credit arm of one banking group to the funding arm of another.

The first objective of the analysis is to cluster the funding and credit nodes of all the different banking groups together in a way that accurately reflects areas of concentration of financial stress. In particular, modules are defined so that stress travels between the members of a module with a greater intensity than it does to the nodes outside the module. For this purpose we use a network clustering technique developed by Rosvall and Bergstrom (2008), henceforth RB. RB's *map equation* determines the most parsimonious yet accurate description of the network that can be used to map the movements of an imaginary traveller, taking account of how likely he is to visit each node. Groups of nodes with long persistence times are clustered

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[1] For more on the evolution of financial markets from a network perspective see Haldane (2009).

together. Because the approach clusters the network using information about flow, the approach has an advantage over generalised modularity approaches (for example Newman (2006), Girvan and Newman (2002) or Blondel, Guillaume, Lambiotte and Lefebvre (2008)) that focus only on pairwise aspects of the link structure.

Success depends upon the proper specification of a transition probability matrix that governs the flow of stress through the system. We define this matrix using data on financial claims between banking entities. Our approach emphasises mismatches between assets and liabilities. Under our specification, the prestige of each module, which is measured by the frequency with which shocks visit the module in a steady state, depends not only on the sum of the gross assets and liabilities of each banking group in the module, but also on the mismatch between liabilities and assets. The modules where financial stress visits the most are those with large and mismatched balanced sheets.

Clustering is in general a difficult numerical problem because of the vast number of modular permutations possible in even a small network. A crucial advantage of RB's approach is that it uses advances in information theory, in particular a generalisation of Shannon's source coding theorem (Shannon (1948)), to simplify the computational burden associated with evaluating all possible clustering arrangements. For this reason, RB's approach is well suited to determine a revealing map of the flow of stress through the international financial network.

Describing the system at a modular level is an important part of our analysis of systemic risk in the international banking network. Given our modular description of the network we can see which countries belong to the same module and hence are most heavily impacted by each other in times of financial stress. We also examine the flow within and between modules. In a safer network, the most important modules will have a lower capacity to transmit financial stress; those modules will act as absorbers. If instead the important modules have a high propensity to transmit contagion, then financial stress is more likely to criss-cross many national boundaries and become truly systemic.

When financial stress crosses many national boundaries it is more problematic. This is in part because different legal systems and political preferences have to be compromised. For example, London School of Economics Law and Financial Markets Project (2009) explain that

Lehman Brothers' global business operated with over 100 data systems that were owned and managed by some of the 6,000 legal entities within the group worldwide. Once the global firm failed, administrators in each country where the firm operated needed to co-operate over sharing the very high cost of running these data systems. Claessens, Dell'Ariccia, Laeven and Igan (2010) and Tucker (2010) emphasise difficulties in international co-ordination over crisis resolution. A corollary is that a network where financial stress can move rapidly to and fro across national boundaries should feature a greater risk of a systemic crisis.

By combining modularity with measures of the probability of contagion at that modular level, we are able to identify when stress can flow freely through the international banking network and when it is likely to be corralled by absorption. As we will see, the financial system may be most vulnerable when multiple large modules are highly contagious.

An interesting by-product of our analysis is that it provides insight into which countries need to co-operate most to police systemic risk in the international banking network. Our specification of modules identifies the countries between which risk flows are most concentrated. If a module is absorbing, then collective action by countries within that module may be sufficient to contain systemic risk. If instead that module were highly contagious, then this suggests that other countries should be involved. An analogy can be drawn to global warming. Since each country's adverse actions spreads negative externalities all over the world, the minimum cluster for dealing with the problem should be the whole world.

2 Previous work

Models of networks for the purpose of analysing systemic risk fall into two categories. One class of models are those aimed at simulating financial stress across the network. These are reviewed in International Monetary Fund (2009). The latest generation of these simulation network models incorporate the lessons of the crisis and feature sophisticated transmission through funding and fire-sale externalities and not just through chains of credit tightening (Gai and Kapadia (2010)). Naturally they require quite a few calibrations and detailed modelling of the behaviour of each node. And the results they report are more in the form of specific experiments.

Our paper falls into another strand of the literature which, rather than simulating particular experiments, aims to summarise relevant features of the network without imposing too many assumptions. Within this subgenre, there are no other papers which allow for both funding and credit channels. Moreover, the two papers which have carried out network-measure based analysis on the international banking network (von Peter (2007) and Kubelec and Sa (2010)) do not consider modular structures.

The relevant features of the network that we seek to summarise relate to connectedness, because that tells us how stress can flow around the system. However, standard measures of interconnectedness do not show large variation over time when applied to financial networks. Measures of the prestige of each banking group do not change very much either during the build-up or in the aftermath to this global banking crisis. If we were to take these standard measures of network interconnectedness at face value, we may be led to conclude that systemic risk in the network is more a question of scale (the total value of claims in the network) rather than about the cross sectional aspects (the distribution of claims across the matrix of bilateral exposures). We might also be led to conclude that developments across these two dimensions are quite independent.

This paper departs from the literature on summarising systemic risk in the international interbank network by allowing for funding and credit channels for the transmission of stress and by deriving modularity from an analysis of the movement of stress across the network. For these reasons, our measure is sensitive to changes in the cross-sectional distribution of claims, as we think it should be. This gives us the power to track when systemic risk in the network is particularly elevated.

3 Funding and credit risk

The current crisis was transmitted between banks both because borrowing banks defaulted and also lender banks cut funding; credit and funding transmission were intertwined. Thus there should be four possible channels between any two different banking groups.

Our solution is to split each banking group into two nodes, one for each side of the balance sheet, so that there are separate funding and credit channels between different banking groups.

To allow for contagion to pass from a banking group's creditors to its funders and *vice versa*, it must be that the two bi-nodes of the same banking group are also connected to each other, as if they were two departments in the same bank, but with an implicit contract. Indeed from the Bank for International Settlements data we know that cross-border intragroup claims account for about a third of all external claims between international banking groups. For some multinational banks, these claims are even priced in an internal market.

The modular structure of the network should depend upon the intrabank links. If a banking group's intrabank links are strong, then contagion can be trapped within that banking group and its important trading partners. At the same time, contagion is discouraged from spreading to the less related parts of the system. So close nodes are pulled together and distant nodes are pulled apart. In this way, allowing for both credit and funding channels inevitably implies modelling the intrabank contract, which in turn leads to a relevant and interesting modular structure for the interbank network.

Denote the set of banking groups (or countries) by G. Formally, there are two types of nodes for each country: bank funding departments and bank credit departments, defined respectively as α_F and α_C for $\alpha \in G$. Let $x_{\alpha_F \beta_C}$ represent the money value of the claim that the credit arm of banking group β holds on the funding arm of banking group α; this term represents the value of loans that banks in country β have made to banks in country α. What follows is a scheme for translating these money values into weighted directed links that indicate the ability of risk to flow through the financial network, and hence determine the path of contagion.

We assume that weights for when contagion travels *between any two banking groups* are given by

$$v_{\alpha_F \beta_C} = v_{\beta_C \alpha_F} = x_{\alpha_F \beta_C}, \text{ for } \alpha \neq \beta, \tag{1}$$

and

$$v_{\alpha_F \beta_F} = v_{\alpha_C \beta_C} = 0, \text{ for all } \alpha, \beta \in G. \tag{2}$$

Contagion can go up or downstream. The value $v_{\alpha_F \beta_C}$ is the weight on the directed link from the credit arm of banking group β to the funding arm of banking group α. This is the pathway for *funding risk* because it relates to the event that banking group β stops lending to banking group α. While the trigger for the financial crisis was the poor performance of securities and loans backed by US mortgages in early 2007, a main channel of transmission was the pressure that this put on bank funding markets. The spread between the three-month Libor and central bank repo rates increased from about 5-15 basis points to above 70.

The value $v_{\beta_C\alpha_F}$ is the weight on the directed link from the funding arm of banking group α to the credit arm of banking group β. This is the pathway for *credit risk* because it relates to the event that banking group α defaults on its loan from banking group β. Since both risks relate to the same nominal contract, the claim that banking group β holds on banking group α, these weights are the same. Of course, this assumes that creditors and funders suffer the same blow when there is a problem with a contract, which may be debatable.

There is no clear-cut way to define the absorptive capacity of risk across funding and credit arms of the same banking group. For now we leave this question open and define the weights on links across arms of banking group α by the parameter w_α:

$$v_{\alpha_F\alpha_C} = v_{\alpha_C\alpha_F} = w_\alpha, \text{ for all } \alpha \in G. \tag{3}$$

Later on we justify a particular choice of weights for these terms.

Using equations **(1)** to **(3)**, the matrix of contagion frequency in our network (the source along the columns, destination along the rows) is the matrix

$$\mathbf{V} = \left(v_{\alpha_J\beta_K}\right)_{\alpha_J,\beta_K}, \tag{4}$$

where $\alpha, \beta \in G$ and $J, K \in \{C, F\}$ and the ordering is by the credit arm of banking group 1, the credit arm of banking group 2, for all countries, and then the funding arm of banking group 1, the funding arm of banking group 2, for all countries. Suppose $|G| = n$. Then $\mathbf{V}$ is a $2n \times 2n$ symmetric matrix.

In order to describe the path of contagion we need to convert these weights into probabilities. Recall that each element v_{ij} of the matrix $\mathbf{V}$ describes the directional weight from j to i; ie, the capacity for stress to be transmitted from j to i. Our premise is that contagion flows out of node j according to probabilities that are proportional to these capacities. So, for example, if $v_{ij} = 2v_{kj}$, then (conditional on its moving out of node j) contagion is twice as likely to pass to i as it is to k. Hence, we convert the weights in $\mathbf{V}$ into probabilities by transforming the matrix $\mathbf{V}$ into the column-stochastic *Markov transition matrix*

$$\Pi = \left(\pi_{\alpha_J\beta_K}\right)_{\alpha_J,\beta_K} = \left(\frac{v_{\alpha_J\beta_K}}{\sum_{i=1}^{2n} v_{i\beta_K}}\right)_{\alpha_J,\beta_K} \tag{5}$$

where $\alpha, \beta \in G$ and $J, K \in \{C, F\}$ and the ordering copies $\mathbf{V}$.

4 Methodology

How do we decide on the best modular structure that fits the network that intertwines funding and credit transmission? At the core of RB's approach is a formula that tells us how efficient any particular modular structure is at describing the path of an imaginary traveller, whom we call Mr Contagion, around the network, given information about the stochastic process that determines his movements. This is their map equation.

RB's idea was to consider the dual problem of compressing data and finding patterns. We want a good map of how risk travels through the network. A good map simplifies away unnecessary details and highlights important ones. In this case the important details are the modules where Mr Contagion is likely to stay confined for extended periods of time once he enters them. The path of Mr Contagion is described by a set of codebooks, one high-level index codebook and then a series of low-level module codebooks. To describe travel between modules always requires use of the index codebook, and travelling within or to a module requires that module's codebook. Each map book contains its own set of names which can be repeated across books but never within. Short names are used first for the nodes most visited.

The advantage of using two codebooks is that it can economise on the amount of information (bits) needed to describe the path of Mr Contagion. By introducing a new module, it is possible to reuse short codenames, which require less bits. However, an additional codeword is needed in the index codebook to identify the new module. There is a trade-off: if there are too few modules then the modular codebooks will need long names, but if there are too many then the index codebook will include too many names. The most efficient balance depends on the frequency with which contagion visit nodes. The important point is that the problem of dividing the nodes into modules is dual to the problem of designing an efficient map.

To derive the map equation formula explicitly, let p_{α_J} be the frequency that the traveller visits node α_J, for $\alpha \in G$ and $J \in \{C, F\}$, and let $q_{i\curvearrowright}$ be the frequency with which module i is exited. These would be measured after the traveller has been moving around the system for a long enough duration that his initial starting point becomes irrelevant. Mathematically, the values

p_{α_J} are computed as the dominant (right) eigenvector of the Markov transition matrix of contagion, Π:

$$\mathbf{p} = \Pi \mathbf{p}, \tag{6}$$

where $\mathbf{p} = [p_1, \ldots, p_{2n}]'$. This measure of eigenvector centrality can be calculated if the Markov transition matrix is irreducible.[2] Furthermore we show later on that as the matrix of contagion on which the Markov matrix is calculated, equation **(4)**, is symmetric, the eigenvector centrality of each node is equal to the share of that node's column sum in the total weight of the matrix. In other words, in our set-up, the prestige of each node α_J can easily be calculated as the ratio of the sum of the weights in column α_J to the sum of all weights in the matrix $\mathbf{V}$.

Given the prestige parameters **p** from equation **(6)**, and an arbitrary modular structure M, with $i = 1, ..., m$ modules, the modular exit frequency is given by

$$q_{i\curvearrowright} = \sum_{\beta_C \in i} \sum_{\alpha_F \notin i} \pi_{\alpha_F \beta_C} p_{\beta_C} + \sum_{\beta_F \in i} \sum_{\alpha_C \notin i} \pi_{\alpha_C \beta_F} p_{\beta_F}. \tag{7}$$

Given equations **(6)** and **(7)**, or any other appropriate expression for these two frequencies, we can follow the procedure outlined in RB and calculate the frequency with which the traveller would need to use module *i*'s codebook. Here, this expression is given by

$$p^i_{\circlearrowleft} = q_{i\curvearrowright} + \sum_{\alpha_C \in i} p_{\alpha_C} + \sum_{\alpha_F \in i} p_{\alpha_F}. \tag{8}$$

Likewise, the value

$$q_{\curvearrowright} = \sum_{i=1}^{m} q_{i\curvearrowright} \tag{9}$$

is the frequency with which the traveller would exit any module, and therefore need the index codebook.

The probabilities $p^i_{\circlearrowleft}$ and $q_{\curvearrowright}$ tell us how often the modular and index codebooks are used. Next we need to know how costly (in terms of bits) it is to access these codebooks. These costs must be based on the optimal assignment of codenames with respect to the usage frequencies of the names in the various codebooks. RB do not need to actually produce optimum codenames for each codebook under every possible partition. Rather, they calculate the theoretical limits for all of the different partitions using Shannon's coding theorem and pick the one that gives the shortest description length. Shannon's coding theorem tells us that when N codewords are used to describe the N states of a random variable z that occur with frequency p_i, the average

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[2] This follows from the Perron-Frobenius theorem for irreducible matrices; see, for example, Seneta (1981).

length of the codeword can be no less than the entropy of z, defined as

$$H(z) = -\sum_{i=1}^{N} p_i \log(p_i). \tag{10}$$

Thus, the minimum description lengths for the index and modular codebooks are given by

$$H(Q) = -\sum_{i=1}^{m} \frac{q_{i\curvearrowright}}{q_{\curvearrowright}} \log\left(\frac{q_{i\curvearrowright}}{q_{\curvearrowright}}\right) \tag{11}$$

and

$$H\left(P^i\right) = -\frac{q_{i\curvearrowright}}{p^i_{\circlearrowright}} \log\left(\frac{q_{i\curvearrowright}}{p^i_{\circlearrowright}}\right) - \sum_{\alpha\in i} \frac{p_\alpha}{p^i_{\circlearrowright}} \log\left(\frac{p_\alpha}{p^i_{\circlearrowright}}\right) \tag{12}$$

where H is the entropy function and Q and P^i denote distributions of usage frequencies of the names in their respective codebooks.

The minimum description length of the random path followed by Mr Contagion when the whole system is organised into a particular structure M with m modules is thus given by RB's map equation:

$$L(M) = q_{\curvearrowright} H(Q) + \sum_{i=1}^{m} p^i_{\circlearrowright} H\left(P^i\right). \tag{13}$$

$H(Q)$ is the frequency weighted average of the minimum length of names in the index guide and $H(P^i)$ is the frequency weighted average of the minimum length of names in the guide to module i. Thus the map equation is the weighted average of minimum name length of the index map book and the module map book.

By following RB's technique and minimising $L(M)$ across all possible structures, we can identify the most efficient description of the network, which may identify potential 'hotspots' of contagion.[3] This does not rule out the possibility that the best description is that all nodes are in one module. Once we know the modular structure of the network, we can use the components of RB's map equation to produce an estimate of the propensity for each module to transmit, or conversely to contain contagion.

The prestige of a module is given by the sum of the prestiges of all the nodes contained in the module. This prestige can be thought of as the frequency with which shocks visit the module. One can think of a single shock or thousands of shocks, each of which can originate at different places and at different times, but which move through the system according to the Markov transition matrix specified in equation **(5)**. Some of these visits will leave the module

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[3] RB have developed an ingenious *greedy algorithm* for minimising **(13)** over large networks. This analysis was conducted using software provided by Martin Rosvall and available at www.tp.umu.se/ rosvall/.

in the next step while others will remain. In other words, the prestige of a module can be divided up into two portions, the part that relates to travel within the module and the part that relates to travel outside the module.

The exit frequency of module i is given by $q_{i\curvearrowright}$ (see equation **(7)**). If a module has a relatively high exit probability (versus non-exit) then the interpretation is that shocks to the module will be transmitted to the rest of the system with high frequency. Whereas, a low exit probability suggests that much of the damage from shocks that reach the module will be absorbed within the module; ie, it is more likely that the damage will be contained.[4]

5 Modelling intrabanking group transmission

In this section, we explain our choice for the weight of intrabanking group transmission, which up until now has been specified as the term w_α in equation **(3)**.

We begin by confirming our earlier claim that the prestige of each node is equal to the shares of each column (or row) sum in the total weight, since that is a key step in our argument. Consider the vector $\Pi\mathbf{z}$, where $\mathbf{z}$ is the $1 \times 2n$ vector of column sum shares in the total weight, with the k^{th} element given by

$$\frac{\sum_{i=1}^{2n} v_{ik}}{\sum_{i=1}^{2n}\sum_{j=1}^{2n} v_{ij}}. \qquad \textbf{(14)}$$

Given the definition of Π in equation **(5)**, the k^{th} element of $\Pi\mathbf{z}$ is

$$\sum_{j=1}^{2n} \frac{v_{kj}}{\sum_{i=1}^{2n} v_{ij}} \frac{\sum_{i=1}^{2n} v_{ij}}{\sum_{i=1}^{2n}\sum_{\ell=1}^{2n} v_{i\ell}} = \frac{\sum_{j=1}^{2n} v_{kj}}{\sum_{i=1}^{2n}\sum_{\ell=1}^{2n} v_{i\ell}} = \frac{\sum_{j=1}^{2n} v_{jk}}{\sum_{i=1}^{2n}\sum_{\ell=1}^{2n} v_{i\ell}}.$$

The last equality follows from the previous one because the row and column sums of the symmetric matrix $\mathbf{V}$ are identical. We have thus shown that $\mathbf{z} = \Pi\mathbf{z}$ and hence $\mathbf{z}$ is the unique vector of prestiges.

Given assumptions **(1)** to **(3)**, and using formula **(14)**, the prestige of the credit arm of the banking group α can be written as

$$p_{\alpha_C} = \frac{1}{2}\frac{\sum_{\beta\in G} x_{\beta_F\alpha_C} + w_\alpha}{\sum_{\beta\in G}\sum_{\gamma\in G} x_{\beta_F\gamma_C} + \sum_{\beta\in G} w_\beta},$$

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[4] Because the system is irreducible there are no ergodic sets, other than the whole system. Thus, Mr Contagion will never be completely trapped in a module.

and that of its funding arm is

$$p_{\alpha_F} = \frac{1}{2}\frac{\sum_{\beta\in G} x_{\alpha_F\beta_C} + w_\alpha}{\sum_{\beta\in G}\sum_{\gamma\in G} x_{\beta_F\gamma_C} + \sum_{\beta\in G} w_\beta}.$$

The total prestige of the banking group α is the sum of the two, or

$$p_\alpha = \frac{\frac{1}{2}\left(\sum_{\beta\in G} x_{\beta_F\alpha_C} + \sum_{\beta\in G} x_{\alpha_F\beta_C}\right) + w_\alpha}{\sum_{\beta\in G}\sum_{\gamma\in G} x_{\beta_F\gamma_C} + \sum_{\beta\in G} w_\beta}. \tag{15}$$

If $w_\alpha = 0$ for all banking groups α, then the prestige of any banking group α is simply the equally weighted share of assets and liabilities of that banking group of the total values in the system. As prestige measures the share of visits that Mr Contagion makes to a node, it does not seem realistic that prestige should depend purely on the relative share of a banking group's assets and liabilities. That would mean for example that a banking group that has \$800 billion of liabilities and \$400 billion of assets will have the same prestige as a banking group whose assets were \$800 billion and liabilities half that while in reality we would expect the first banking group to have more lure for stress.

The role of the term w_α in the more expanded expression **(15)** is to improve on the benchmark by shifting prestige from nodes where intrabanking transmission is low to nodes where intrabanking transmission is high. But much depends on what exactly determines w_α. Our starting point is that the extent of balance sheet mismatch should matter in determining prestige, so that a banking group which has a large interbank funding requirement relative to its interbank assets receives more contagion. For consistency, we choose to make each pair of intrabanking group contracts equal to the total liabilities of that banking group in the whole system:

$$w_\alpha = \sum_{\beta\neq\alpha} x_{\alpha_F\beta_C}. \tag{16}$$

Then the prestige of banking group α is

$$\begin{aligned} p_\alpha &= \frac{\left(\frac{1}{4}\sum_{\beta\in G} x_{\beta_F\alpha_C} + \frac{3}{4}\sum_{\beta\in G} x_{\alpha_F\beta_C}\right)}{\sum_{\beta\in G}\sum_{\gamma\in G} x_{\beta_F\gamma_C}} \\ &= \frac{1}{2}\frac{\left(\sum_{\beta\in G} x_{\beta_F\alpha_C} + \sum_{\beta\in G} x_{\alpha_F\beta_C}\right)}{\sum_{\beta\in G}\sum_{\gamma\in G} x_{\beta_F\gamma_C}} + \frac{1}{4}\frac{\left(\sum_{\beta\in G} x_{\alpha_F\beta_C} - \sum_{\beta\in G} x_{\beta_F\alpha_C}\right)}{\sum_{\beta\in G}\sum_{\gamma\in G} x_{\beta_F\gamma_C}}, \end{aligned}$$

implying that frequency of Mr Contagion visiting is greater, the more gross assets and liabilities the banking group has compared to the other groups, and over and above that, if it has large net liabilities. The weight on the mismatch component is a half of the weight on the gross position benchmark.

In principle this could be refined. For example the transmission between the two halves of the bank could be made to depend on more specific properties of each banking group. But that would place undue emphasis on our ability to measure the true structure of each banking group's assets and liabilities. Our assumption should be seen in the spirit of an uninformed prior. This is the kind of structure that could be imposed by a regulator designing a system for the optimal distribution of contagion, if that regulator did not want to rely on any other data on each banking group other than their total asset and liabilities to other banking groups.

6 The data

We measure the claims held by each country's resident banks on each other country's resident banks as reported in the Bank for International Settlements (BIS) locational by residency statistics. In this data set, both domestically owned and foreign-owned banking offices in the reporting countries record their on balance sheet positions on other countries. These data are consistent with the residency concept of national accounts. It features all banks in each country with significant external claims, and includes the US investment banks that were protagonists of the crisis. The BIS guide to these statistics (Bank for International Settlements (2010)) contains a list of the reporting entities in each country in the annex. Another advantage of this bilateral data set is that it contains many types of financial claims that carried contagion during this crisis. As well as standard loans and deposits, banks report on sale and repurchase transactions, certificates of deposits, financial leases, promissory notes, subordinated loans, debt securities, equity holdings and participations. Debt securities would include funding through trust preferred securities and asset-backed securities, as long as issuer and holder were reporting banks residing in different countries. As far as we know, there is no publically available bilateral data set at the level of individual banking entities or even consolidated banks.

As comprehensive as the data are, they may not capture all the channels of contagion that mattered for this crisis. For example banks' exposure to risk through derivatives such as credit instruments or swaps and futures (Segoviano and Singh (2008)), are difficult to measure. Also absent are off balance sheet positions through special bank-sponsored vehicles. Finally we do not have complete data on other non-bank financial institutions that became commingled with banks in the build-up to the crisis, the so-called shadow banking system. These missing links mattered in transmitting the crisis. Also European banks funded themselves through purpose-built, and supposedly independent off balance sheet vehicles that lent long term back to their bank creators. These conduits funded these investments by issuing and rolling over shorter-maturity paper, sometimes using a complex repackaging of the securitised assets of their creators as collateral (Longstaff (2010)). Even though losses only appeared in some of the components of composite assets, these instruments were sufficiently difficult to disentangle that investors' confidence in the whole market was shattered across the board (Coval, Jurek and Stafford (2009)). As the banks that created these damaged vehicles took them back on to their own balance sheets, they acknowledged their exposure to this correlated risk. Other non-bank financial companies, such as US money market mutual funds, had purchased many of these assets (Bertaut and Pounder (2009)) and other non-bank financials, such as monoline insurers, had guaranteed payments and both came to be embroiled in the crisis.

These are then channels of contagion between banks through common participation in markets with shadow banking institutions rather than through direct claims held on each other. They cannot be incorporated because we do not have definitive data on these conduits' links with their bank creators, their residency, a clear idea of what entities they ultimately borrowed from and who underwrote the risks in these transactions. Although we cannot fully account for off balance sheet derivatives and the shadow banking sector, our analysis retains its validity provided the patterns described by the data we do have would resemble the complete network with the missing data closely enough.

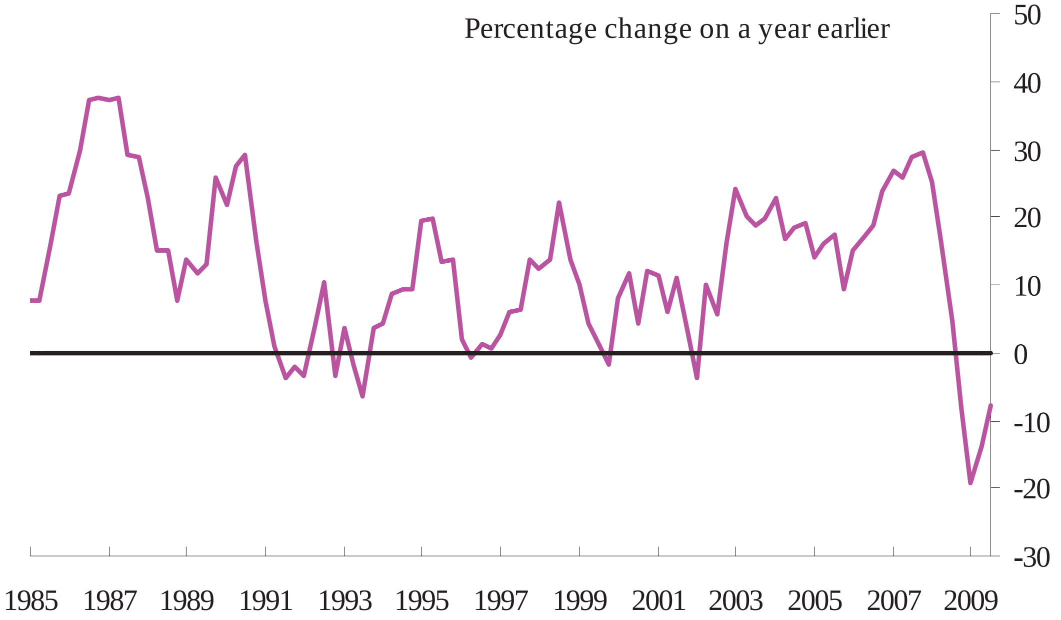
We include the following 21 reporting countries in our network: Austria, Australia, Belgium, Canada, the Cayman Islands, Switzerland, Germany, Greece, Denmark (excluding Faeroe Islands and Greenland), Spain, Finland, France (including Monaco), United Kingdom (excluding Guernsey, Isle of Man and Jersey), Ireland, Italy, Japan, Luxembourg, Netherlands, Portugal, Sweden, and the United States. Among these are those countries which are the most important to the network and many of the countries excluded do not have complete series. All together our subsample captures about 73% of total reporting banks' claims on banks in all vis-à-vis countries and the growth rate of the total claims in our subset is very similar to the growth rate in the total available to the BIS. We may have left ut some country banking groups, such as in for example Asia, who play a greater part in determining interconnectedness than is belied by their relative share of claims.

We apply our analysis to a sample starting in 1985 Q1 and ending in 2009 Q3. There are a few missing claims in the early data but we filled those in using the same proportion as the most complete data set that we have (2000). No one claim we filled in was more than 0.4% of the total value of all claims, and most were smaller than 0.1%. There were only eight claims filled in any year at most.

7 The international interbank market from 1985 to 2009

Before we go on to analyse the cross-sectional distribution of the interbank network, it is worth reviewing the important developments in this market. Chart 1 plots the annual growth rate of all claims in our data over this period.

Chart 1: BIS external claims of banks on banks in a sample of 21 countries



Source: Bank for International Settlements, Locational by Residence data. The countries are Austria, Australia, Belgium, Canada, the Cayman Islands, Switzerland, Germany, Greece, Denmark (excluding Faeroe Islands and Greenland), Spain, Finland, France (including Monaco), United Kingdom (excluding Guernsey, Isle of Man and Jersey), Ireland, Italy, Japan, Luxembourg, Netherlands, Portugal, Sweden and the United States.

The international interbank market was growing in 1985 when our sample begins. The market grew strongly until 1987, and then after a brief pause following the stock market crash in 1987, picked up speed again to finish the decade in strength. According to Bernard and Bisignano

(2000), an important driver of this expansion were Japanese banks which were channelling surplus domestic funds onto world markets. European banks also became increasingly active in cross-border lending over this period as foreign exchange controls were removed in France and Italy, and as the prospect of greater financial and trade integration in the region loomed. It might have also mattered that many countries independently placed lower capital requirements on interbank lending than on commercial credits during the 1980s and that many banking centres liberalised their domestic financial regulations.

This first boom petered out by the end of the decade. The United States, Japan, Sweden and Finland suffered a sequence of domestic banking crises and world growth slowed down. There was also turbulence associated with the speculative attacks on the European Exchange Rate Mechanism. International interbank flows remained subdued until 1994. Japanese banks in particular began to withdraw lending to other major international banks from 1989 Q4, although as we shall see they did soon start to lend to other Asian country banks.

The international interbank market revitalised again from 1994 until 1997. Bernard and Bisignano (2000) explain how this second boom was related to an excess of liquidity and low market interest rates, just as in the build-up to the recent crisis. European banks, especially Swiss banks, increased their share of this market. Many funds were channelled through offshore centres, in another parallel with the more recent build-up (Dixon (2001)). Some of these funds went to Asian economies, such as South Korea, although they are not in the sample (Kaminsky and Reinhart (2000)).

The second growth spurt in our data came to an end with the Asian crisis (1997 and 1998) and the collapse of Long Term Capital Management (1998). Once growth was halted, the rise in US interest rates in 1997, fears over the costs of European economic monetary union and deleveraging of earlier excesses combined to enforce a slow down. The interbank market grew at low or negative quarterly rates until 2002.

When the market picked up again, it grew fast and for a long time. This was the long boom which led up to the current crisis. Various Bank for International Settlements Quarterly Reviews over this period cite and analyse the investments of Asian economies, petro-dollars, the role of offshore financial centres and hedge funds, and more generally excess liquidity as causal factors (McGuire and Tarashev (2006)). Importantly, many countries' banks shared in this expansion, and hence it can be thought of as a truly international expansion (Claessens *et al* (2010)).

As we know, this third boom ended some time between 2007 and 2008. Once the international banks in our groups began to suffer losses on investments with third parties (in the US sub-prime market), they cut back from lending to each other. While this had happened at least twice before since the mid-1980s, the drop in interbank lending since 2008 is remarkably abrupt: in just over a year annual growth decelerated from 30% to -20%!

8 The results

As a first step we use the data from 1985 Q1 to demonstrate the process by which the original network is split into credit and funding arms and then modularised. Chart 2 shows the network before splitting the funding and credit arms of each banking node and before allocating country banking systems into modules. This is the matrix of international banking exposures, straight from the BIS database. Chart 3 plots the network after splitting along the lines of equations **(1)** to **(2)**. The connections between different banking groups are balanced against strong connections across the balance sheet of the same banking group. The forces that transmit funding risk take equal place alongside credit risk channels. With this richer interaction, naturally, the picture becomes more complicated.

The benefit of modularity is shown in Chart 4 where we see the network structures after applying the map equation algorithm to the split system.[5] In Chart 4 the area of each vertex reflects the prestige of that module, which is the sum of the prestige of its members ($\sum_{\alpha_J \in i} p_{\alpha_J}$ for each module i). The prestige of a module is the probability that Mr Contagion will be found in that module, but that does not tell us if he is then about to stay or go. That information is conveyed by the area of the outer ring which shows the probability of leaving ($q_{i\curvearrowright}$ for each module i) and the area of the inner circle which shows the likelihood of staying ($\sum_{\alpha \in i} p_\alpha - q_{i\curvearrowright}$

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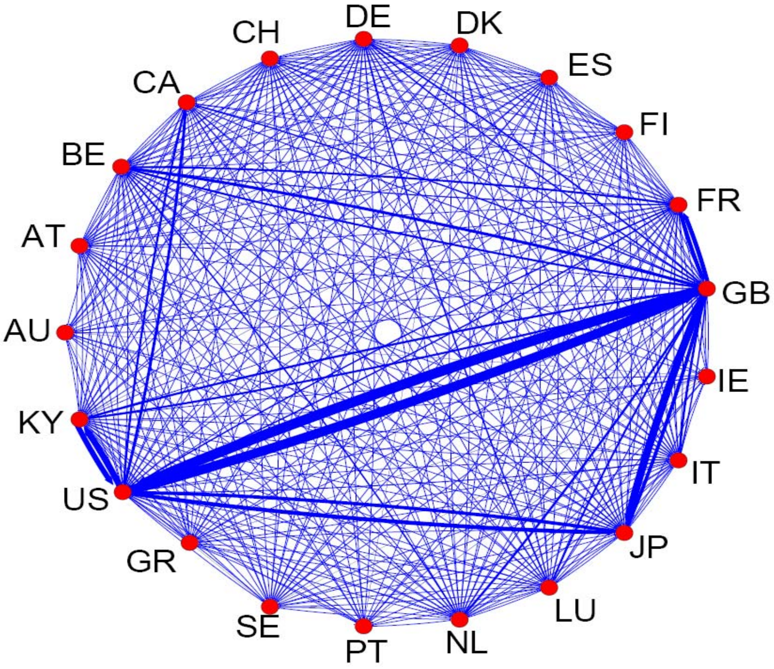
[5] The diagrams presented in Charts 4, 5 and 8 were formed using Martin Rosvall's Map Generator tool, which is available at www.mapequation.org. The options we chose were as follows: placement/links (125); type (circle tree); colour and size tools (absolute scale); label size (root, min 15, and max 20); link size (root, min 0, and max 25); module size (root, min 0, and max, 50); module colour range (linear); link colour range (linear); absolute scale module reference (20%) and link reference (3%). If the difference between the prestiges of the two largest modules was less than 5 percentage points, more than one module is in the centre of the circle.

Chart 2: Nodes before split (1985 Q1)

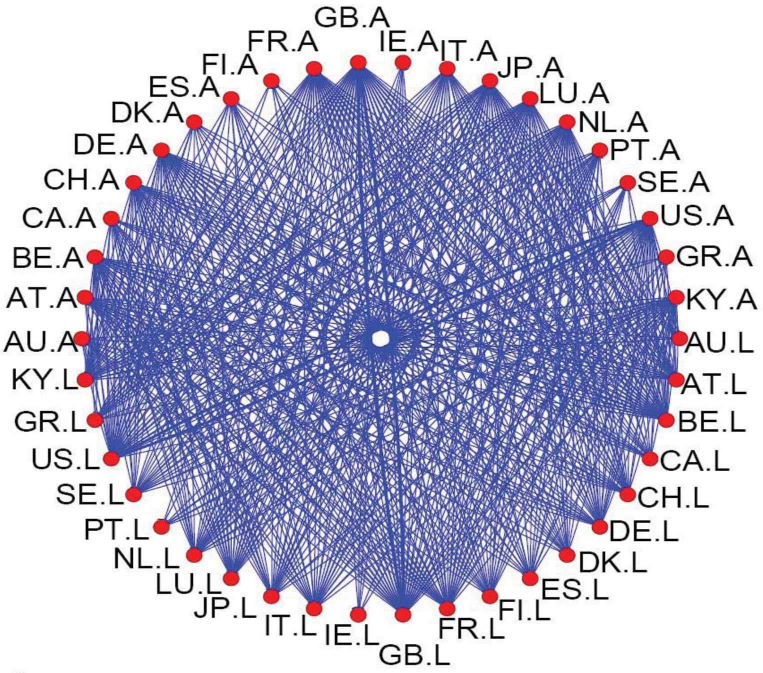


Source: Bank for International Settlements, Locational by Residence data. Austria (AT), Australia (AU), Belgium (BE), Canada (CA), the Cayman Islands (KY), Switzerland (CH), Germany (DE), Greece (GR), Denmark (DK), Spain (ES), Finland (FI), France (FR), United Kingdom (GB), Ireland (IE), Italy (IT), Japan (JP), Luxembourg (LU), Netherlands (NL), Portugal (PT), Sweden (SE) and the United States (US).

). The arrows out of each module further divide up the probability of leaving among the different destinations — the thickness of the arrow going from module i to module j ($i \neq j$) is proportional to the probability that Mr Contagion travels from module i to module j. In all cases, a country's funding arm and credit arm are clustered together into the same module and hence only country labels are used in the chart (this is also true in all of the charts that follow).

In 1985 Q1, the United States formed the most prestigious module with the Cayman Islands. Mr Contagion spends about 25% of his time there. As we shall see these two banking groups remain together for the whole sample, reflecting the fact that the Cayman Islands is an offshore centre for US banking. The IMF recently estimated that 57% of the assets of the Cayman Islands banking system are overnight sweep accounts in branches of US banks (International Monetary Fund (2009)). But in this crisis, contagion could well have traversed this apparently innocuous route — Cayman Islands residents were large foreign holders of private-label US

Chart 3: Nodes after split (1985 Q1)



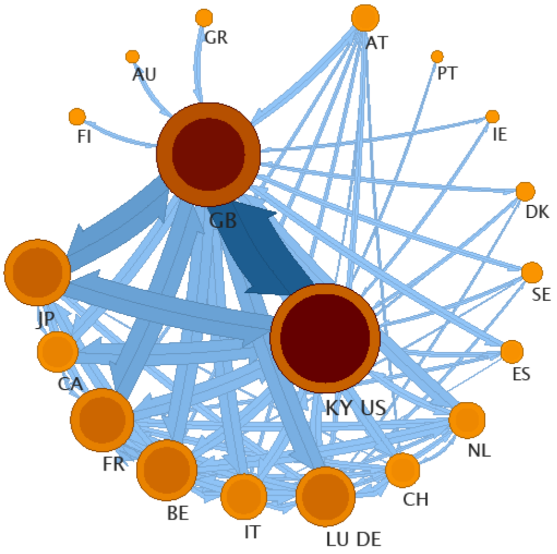
Sources: Bank for International Settlements, Locational by Residence data and own calculations. Austria (AT), Australia (AU), Belgium (BE), Canada (CA), the Cayman Islands (KY), Switzerland (CH), Germany (DE), Greece (GR), Denmark (DK), Spain (ES), Finland (FI), France (FR), United Kingdom (GB), Ireland (IE), Italy (IT), Japan (JP), Luxembourg (LU), Netherlands (NL), Portugal (PT), Sweden (SE) and the United States (US).

mortgage-backed securities leading up to the crisis (Lane and Milesi-Ferretti (2009)).

Then there is another module containing only the United Kingdom which is nearly as prestigous and is where Mr Contagion spends about 23% of the time. It turns out that the UK banking group is always in the most prestigious module or the second most prestigious module for the entire sample, no doubt given its role as a host to many foreign-owned banks as well as the international nature of its own banks. After these two large modules, come four others with much smaller prestige. Japan, Belgium, France and Germany and Luxembourg together, have prestige scores ranging from 9 to 7%. At the other extreme, there are seven modules that have prestige under 1%. Apart from the US-Caymans and Germany-Luxembourg modules, all countries are in their own modules.

It is perhaps not surprising that Luxembourg forms a module with Germany. Distance and

Chart 4: Modular network (1985 Q1)



Sources: Bank for International Settlements, Locational by Residence data and own calculations.

trade can matter it seems in international banking. But the modular structure also reveals for example that though they held important claims on each other, Canada and the United States were not in the same module in 1985 Q1. Presumably at that moment in time US banks and Canadian banks were less linked together than the United States is with the rest of the network. As we shall see also in later periods, modules are sometimes formed by banking groups with geographical or historical links, but not always. Canada and the United States were briefly in the same module during the late 1990s.

Note also that more prestige does not necessarily imply more contagion. The UK module has less prestige than the US-Caymans module. Yet the UK module is the more contagious of the two: the UK module has a larger outer ring and on average larger arrows flowing out than the US module.

Bearing this in mind, we can now survey similar diagrams for the whole sample. We provide diagrams for quarters where there is significant change in the modular structure in the panel in Chart 5. The scaling of the arrows and the circles are fixed across all of the diagrams so that their respective areas are comparable across time. If at one time, a prestigious module is absorbing, the circle will be large and within that, the inner circle will dominate and the outer ring will be narrow. If at another moment, the module is as prestigious but more contagious, the outer ring will be thicker taking up more of the area of the circle of the same size. Small arrows and circles are not shown. Table 1 in the Appendix gives the numbers for prestiges along with the exit frequencies of each module for each diagram in Chart 5. This is the total area of all the outer rings or of all arrows. Numbers for the complete sample are available upon request. Table 2 summarises the changes in modular structure over the whole sample period.

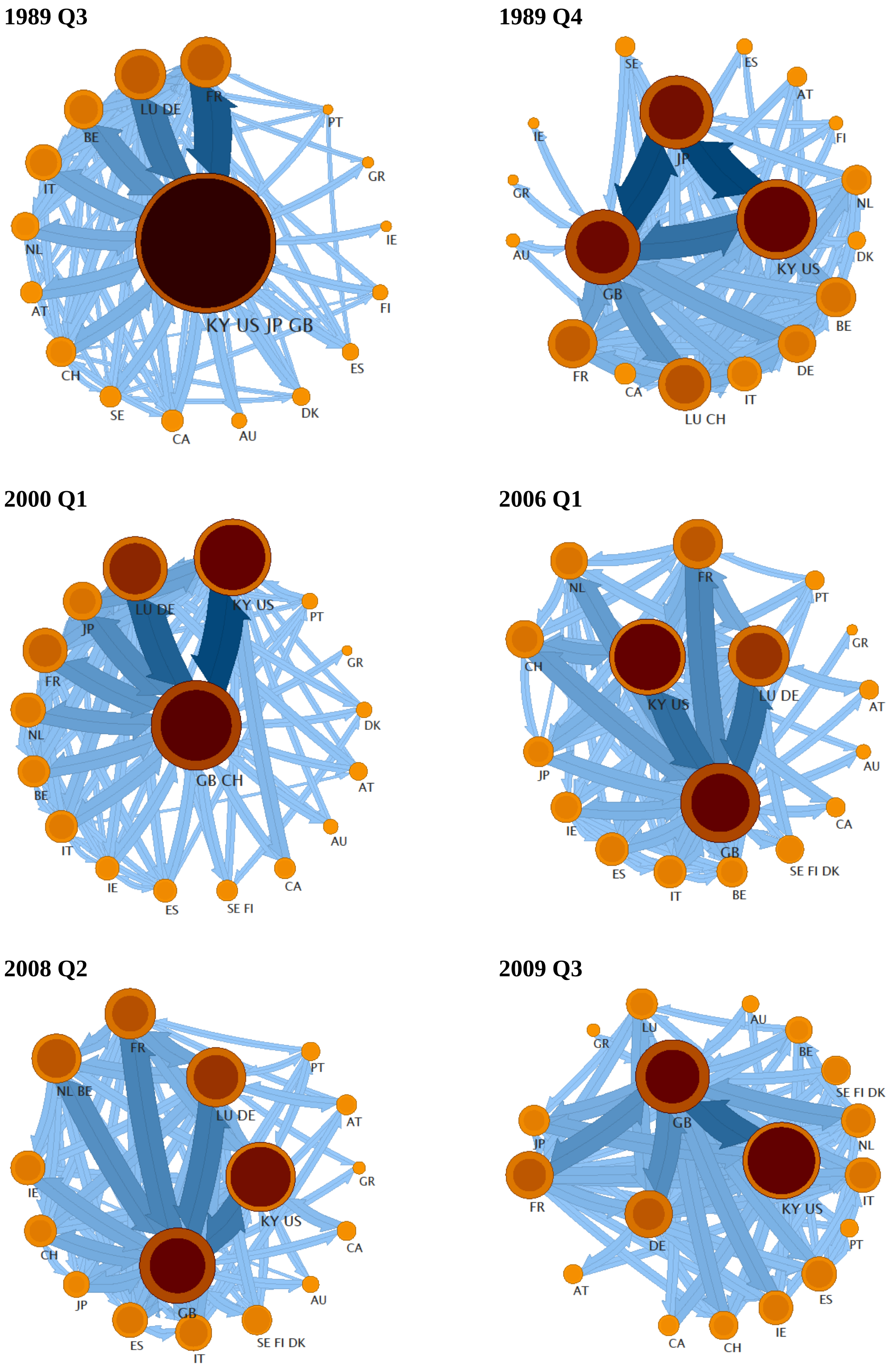
The first story to highlight is the internationalisation of Japanese banks over the second half of the 1980s and the effect this had on the network. Japanese banks start off in their own module in 1985 with a prestige of just over 9%. Their prestige score increases until 1986, when Japanese and UK banks coalesce into a single module with the most prestige. By the end of 1987 this module has combined with the US-Caymans module to form a giant hub in the network. That hub continues to grow until its prestige reaches 63% in 1989 Q3. Crucially for us, that module is quite absorbing. The outer ring takes up 10% of the frequency of the whole network and so once Mr Contagion arrives in this megamodule he is five times more likely to stay there than leave.

As we pointed out earlier, this great drive of the Japanese banks into the international banking market peaked in 1989 Q3. The impact of their retreat on our network is to shatter the large module; see Chart 5, panel 1989 Q4.

The next phase of growth began in the early 1990s when European banks, especially Swiss banks, played a key role. Switzerland, which previously had only been attached to Luxembourg, enters into a module with the United Kingdom. See Table 2a, column 1992 Q3. Also in this period, Germany and Luxembourg form a union, which as we shall see persists through the crisis. In 1997 Q1, Finland and Sweden combine. A few years later in 2000 Q3, Denmark joins them to form a Scandinavian bloc. In 2005 Q1, Belgium and Netherlands merge although, briefly. And although France does not form a module with another country, its prestige steadily rises over this period.

The advance of the European banking groups meant that prestige became more evenly distributed among the larger modules. In 2000 Q1, the US-Caymans module had nearly as

Chart 5: Modular networks



Sources: Bank for International Settlements, Locational by Residence data and own calculations.

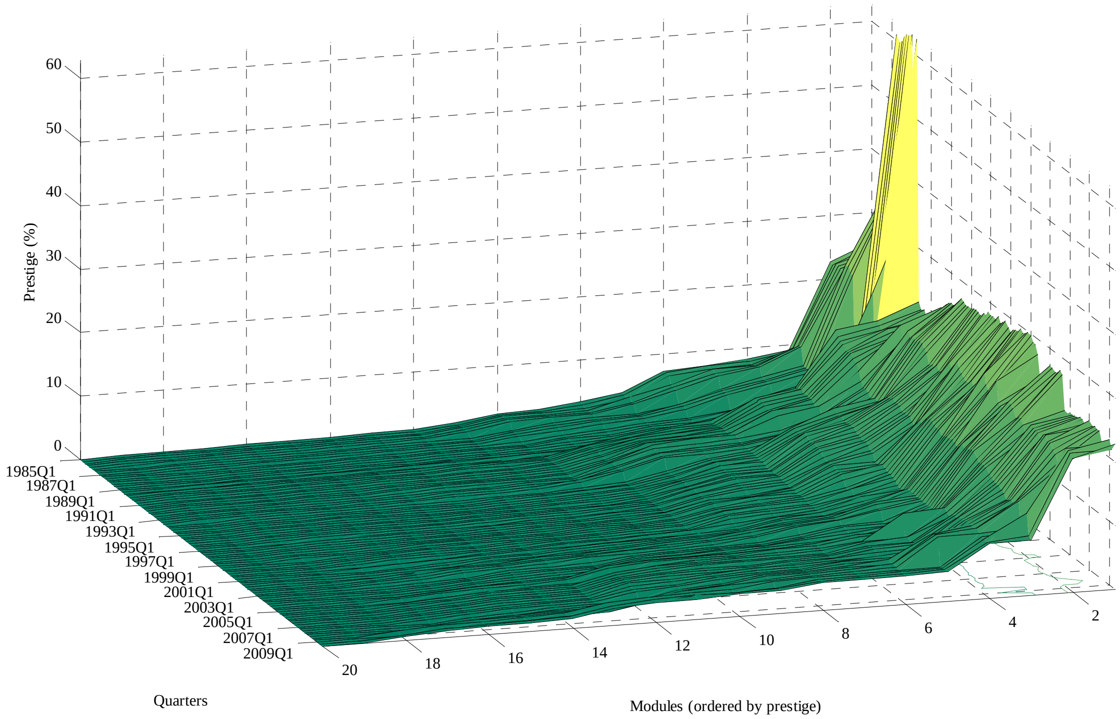
much prestige as the UK and Swiss module (around 25%), and there are now eight modules with prestige above 3% compared to five in 1989 Q3. In general, the modular network is more interconnected, with large arrows from prestigious modules, and relatively larger outer rings (see also Table 1).

A further, bigger shift towards this pattern happens around 2006 Q1 when Switzerland drops out of the largest module with the United Kingdom. As a consequence, the two most prestigious modules lose out to the chasing pack of slightly smaller hubs. There are now eleven modules with prestige above 3%. The network now has less propensity to absorb contagion when compared to the turn of the century and certainly when compared to 1989. In this contagious network, the United Kingdom is singled out as the central hub in this metropolis where stress arrives and is likely to be sent out again to many destinations. A year or so later, stresses from the US sub-prime market began to make themselves felt on international banks. Tellingly, the network remained in this contagious state right up until 2008 Q2, just before the collapse of Lehman Brothers.

Indeed the system remains in a broadly contagious state even until the latest date (2009 Q3). In Chart 5, we can see that the areas taken up by the blue arrows, or the outer rings, in the panel for 2009 Q3 are relatively large compared to a decade or two decades earlier. Despite the massive retrenching during these crisis years, the network at this date had not returned to how it was in 2000 and is not that different to its state when the sub-prime crisis struck. Abstracting from the question of the average quality of investments, that is on cross-sectional grounds alone, one could conclude that financial stress could still be transmitted rapidly around the international banking network.

To see this pattern of change in a more concise form, Chart 6 plots the density of prestige across modules. The chart shows clearly how that density goes from having a steep slope in 1989 Q3 to what looks more like a mountain with a flat cliff by the end of the sample. The flattening happens since 2000 and especially in 2006 reflecting the emergence of multiple important modules in the build-up to the current crisis. Using a country by country data set, Schiavo, Reyes and Fagiolo (2009) also find that the leading financial centres intermediate a large share of asset trade in 2004, much more than they do for goods trade. We have been able to show that the role of hubs has changed over time.

Chart 6: The density of modules prestige



Sources: Bank for International Settlements, Locational by Residence data and own calculations.

8.1 Tracking contagion over time

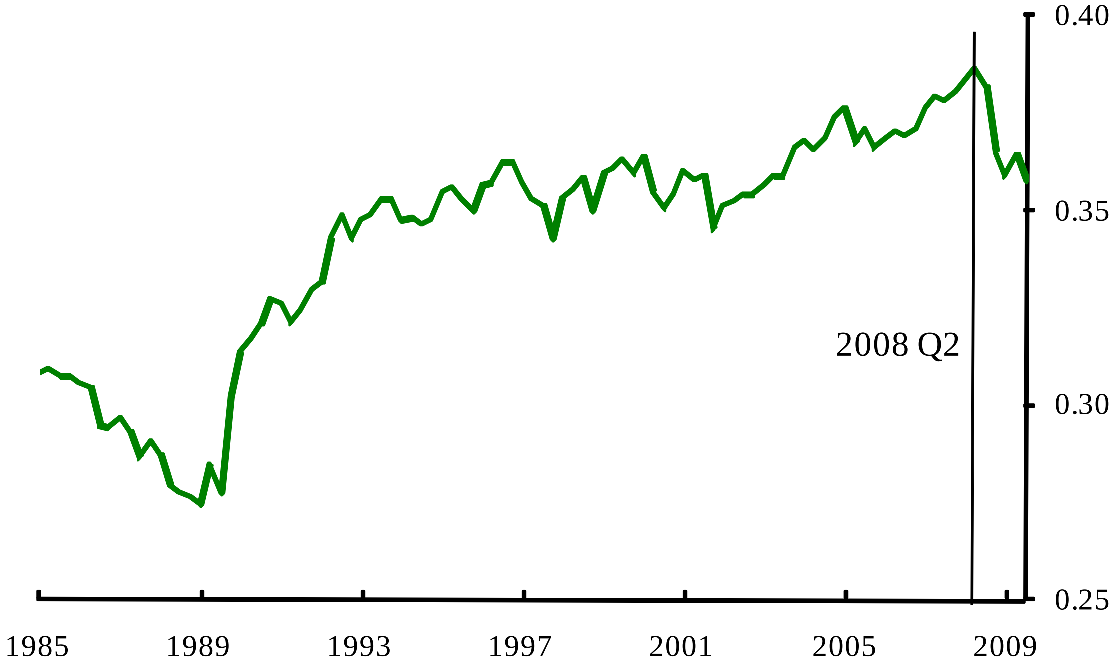
Comparison of the maps in Chart 5 give us a visual indication of how the contagious properties of the international banking network change over time. However, it would be useful to have a simple, quantitative measure. For a given modular structure the measure $q_{\curvearrowright}$ defined in equation **(9)** tells us the fraction of time that a shock travels between modules. This gives us a sense of how broadly contagious shocks are. However, values of this measure are not easily comparable across time periods with different modular structures. Increases in clustering associated with a different optimal modular structure necessarily result in reductions in $q_{\curvearrowright}$ as broader system-wide contagion is internalised into a module. However, it is not appropriate to say the new network is less contagious.

In order to determine whether the international banking network is becoming more or less contagious over time one needs to select a benchmark modular structure and compute $q_{\curvearrowright}$ over time holding the benchmark structure fixed. This tells us whether the amount of system-wide contagion increases or decreases over time and gives additional insight into why changes in modular structure are produced by the map equation.

A natural candidate for this benchmark modular allocation is the one selected by the map equation algorithm in 1989 Q3. The GB-US-KY-JP module had about 63% prestige and an exit probability of 10% in 1989 Q3. This is the largest module of the sample period. By applying this modular structure to the rest of the sample we can see how much of the contagion between these major financial centres spread to other countries over time.

The results are shown in Chart 7. There we can see evidence of the increased capacity of shocks that originated in the GB-US-KY-JP module to be pandemic. The amount of contagion flowing outside the fixed modules increased by over 10 percentage points from 1989 to peak in 2008 Q2, just before the default of Lehman Brothers. The contagiousness of the benchmark structure has fallen since the default of Lehman Brothers, but still remains at a high level relative to the late 80s.

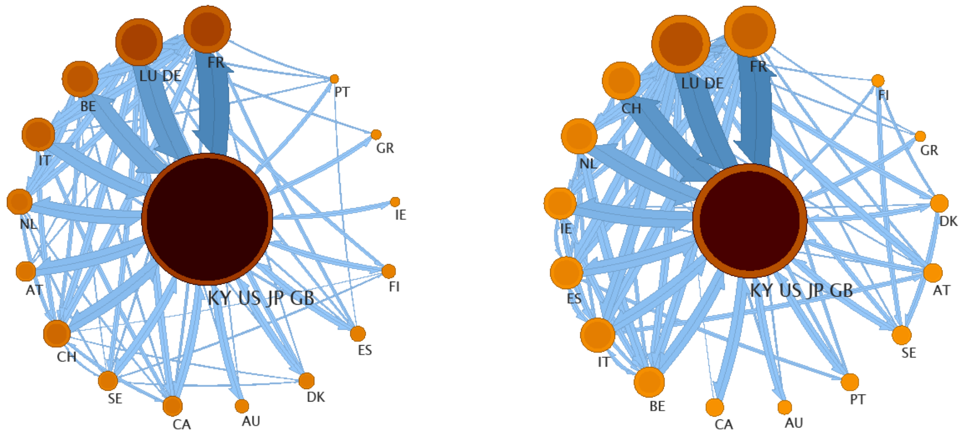
Chart 7: Sum of contagion across modules (imposing 1989 Q3 modular structure)



Sources: Bank for International Settlements, Locational by Residence data and own calculations.

As a final illustration, Chart 8 compares the 1989 Q3 diagram to the same 1989 Q3 modular structure applied to the data on 2008 Q2. This counterfactual modular description was rejected by the map equation for that later date. Note first that the traffic using the blue arrows to leave this counterfactual module are thicker relative to the inner prestige circle than they were in 1989 Q3. We can see that the outer ring of the large module in 2008 Q2 occupies a larger share of the circle compared to the share that it takes in the 1989 Q3 data. This large module would have had an exit probability of 11% but with much less prestige (42%), meaning that if Mr Contagion were to arrive at the 2008 Q2 module, then he is about three times as likely to stay as to leave. Remember that in 1989 Q3 he was five times more likely to stay than leave. The total area of all blue arrows is also greater in the counterfactual 2008 Q2 case, 37% greater in fact, implying that there would be more contagion in the network as a whole also. For all these reasons, the map equation rejects the possibility of the large GB-US-KY-JP module that it selected 20 years earlier for 2008 Q2 because that grouping would have not been able to contain contagion sufficiently. In summary, the network was more broadly contagious in 2008 Q2 than 1989 Q3, and that is revealed to us by this shift in optimal modularity.

Chart 8: The 1989 Q3 modules imposed on the 1989 Q3 data (left) and on the 2008 Q2 data (right)



Sources: Bank for International Settlements, Locational by Residence data and own calculations.

8.2 Robustness checks

Before concluding, it is worth mentioning some of the robustness checks we carried out on our data. First we employed the map equation on the data set before splitting into funding and credit arms to see if that could generate some interesting modular structure without splitting out these two channels that allow for intrabank claims. But the algorithm always only reported one large module suggesting that these new mechanisms are important to understanding contagion.

As another check, we compared the map equation with what should be the leading contender, a version of Girvan and Newman (2002)'s popular modularity function adapted for weighted directed networks by Arenas, Fernández, Fortunato and Gómez (2008):

$$Q(\mathbf{C},\mathbf{V}) = \sum_i \sum_j \left[\frac{v_{ij}}{\left(\sum_i \sum_j v_{ij}\right)} \delta(C_i,C_j) - \frac{\left(\sum_j v_{ij}\right)\left(\sum_i v_{ij}\right)}{\left(\sum_i \sum_j v_{ij}\right)^2} \delta(C_i,C_j) \right] \qquad \mathbf{(17)}$$

where $\mathbf{V} = (v_{ij})_{i,j}$ is the given weighted value matrix of a weighted directed network, C_i denotes the module that node i belongs to and $\delta(C_i,C_j)$ is the kronecker delta which takes a value of 1 if i and j are in the same module, and 0 otherwise. This function can be maximised by the modularity choice $\mathbf{C} = (C_1,\ldots,C_{2n})$. The first term in the square brackets is the value of all links inside modules divided by the value of all links in the whole matrix. The idea is that the best modular description should maximise the weight of links within modules. But there has to be a counterweight, otherwise the best description would trivially be one module. Girvan and Newman (2002)'s chosen counterweight is captured in the second term, the expected value of all links inside modules for the same modular structure, assuming that the in and out links from each node in the same module are randomly and independently reassigned.

Combining expression **(5)** with our earlier result that the prestige of each node is equal to the shares of each column (or row) sum in the total weight, we can rewrite **(17)** in terms of transition probabilities and prestige:

$$\begin{aligned} Q = {} & \sum_{\alpha\in G}\sum_{\beta\in G} \pi_{\alpha_C\beta_F} p_{\beta_F}\,\delta\left(C_{\alpha_C},C_{\beta_F}\right) + \sum_{\alpha\in G}\sum_{\beta\in G} \pi_{\alpha_F\beta_C} p_{\beta_C}\,\delta\left(C_{\alpha_F},C_{\beta_C}\right) \\ & - \sum_{\alpha\in G} p_{\alpha_C} \sum_{\beta\in G} p_{\beta_F}\,\delta\left(C_{\alpha_C},C_{\beta_F}\right) - \sum_{\alpha\in G} p_{\alpha_F} \sum_{\beta\in G} p_{\beta_C}\,\delta\left(C_{\alpha_F},C_{\beta_C}\right). \end{aligned}$$

Using the fact that the sum of prestige is equal to one ($\sum_{\alpha\in G} p_{\alpha_C} + \sum_{\alpha\in G} p_{\alpha_F} = 1$),

$$\begin{aligned} Q = {} & \sum_{\alpha\in G}\sum_{\beta\in G} \pi_{\alpha_C\beta_F} p_{\beta_F}\,\delta\left(C_{\alpha_C},C_{\beta_F}\right) + \sum_{\alpha\in G}\sum_{\beta\in G} \pi_{\alpha_F\beta_C} p_{\beta_C}\,\delta\left(C_{\alpha_F},C_{\beta_C}\right) \\ & + \sum_{\alpha\in G} p_{\alpha_C} \left(1 - \sum_{\beta\in G} p_{\beta_F}\,\delta\left(C_{\alpha_C},C_{\beta_F}\right)\right) + \sum_{\alpha\in G} p_{\alpha_F} \left(1 - \sum_{\beta\in G} p_{\beta_C}\,\delta\left(C_{\alpha_F},C_{\beta_C}\right)\right) \\ & -1. \end{aligned}$$

The first two terms are the expected frequency of travel within modules, or the sum of the areas of the inner rings in Chart 5. This can be trivially maximised at a value of one by having one large module. The second two terms (minus one) are Girvan and Newman (2002)'s counterweight. Each represents the sum of the prestige of each node multiplied by one minus the sum of the prestige of other linked nodes in the same module, and as such can be considered as an estimate of the exit frequency of the modules.

We are now in a position to understand the differences between the map equation and this approach. The map equation also trades off internal travel frequencies against exit frequencies, using information theory to weigh up the cost of either. But the map equation crucially uses the actual exit frequencies of the whole network, whereas this function uses an estimate derived purely from the prestige of each two possible modular partners.

As a final test, we used an algorithm provided by Blondel *et al* (2008) to optimise over function **(17)** on our data set with split funding and credit arms. But the best description was always found to be the uninformative modular pattern where only the funding and credit arm of the same banking group were in each module; modules were equivalent to countries.

To sum up, we found several reasons to prefer the map equation over the leading contender as a tool to analyse modularity on our data set. First, the map equation uses more precise information on financial stress. Second it estimates on the basis of movement in the whole network, not just by calculating pairwise comparisons of each two nodes in a likely module. And third, it delivers informative results.

Another important check was to do with consolidation. As our aim is to understand how financial stress moves around the whole system, the nodes in our network are defined by where banks book their business. In other contexts it might be more appropriate to use a different consolidated data set where banks are allocated by ownership rather than residence (McGuire and von Peter (2009)). We applied our method to a network of consolidated banking groups, available only for a smaller sample, and found a similar pattern for changes in the modular structure, although there were fewer multicountry modules. The additional fragmentation may have been due to the greater data problems in constructing the consolidated data set.

One obvious extension to our paper is to allow for country banking groups to hold claims on each other's and their own non-bank sector. This would build in another very important aspect to the crisis, where banking groups were afflicted not because they had direct claims on each other but because they shared the same credit and funding markets with non-bank entities. For example, if one banking group sells its assets on another country's banking group then that would put pressure on all other banking groups that hold assets on that same non-bank sector. In our initial experiments with introducing non-banks we only found modularity along the lines of the countries. The reason is that the claims held by a banking group on its own non-bank sector are much larger than its other links. This can be overcome by additional parameterisations that downweight links with non-banks, but we have no justification for choosing those parameters.

9 Concluding remarks

If financial stress can be contained within a few countries, it can be more easily dealt with. When the network is so interconnected that stress criss-crosses many national borders, it becomes truly systemic. In these circumstances, resolution is more complicated, the probabilities of default are higher and the losses given default are larger.

The clustering analysis performed in this paper provides insight into when shocks that hit the international banking network might be expected to stay contained within a few countries and when they are likely to spread globally. In cases where stress is likely to travel within groups of countries these countries are clustered together. However, this is not an exact science. The modular structures we compute are based upon a belief that stress flows around the network in a way that is proportional to our estimates of international claims and liabilities, and this can only be approximately true. That said, it is encouraging that our clustering analysis captures well-known changes in the international banking landscape that have occurred over the past quarter century.

Changes in modular structure only tell part of the story. We also examine the extent to which modules transmit stress and how this changes over time. Using a fixed modular structure that combines the major financial centres as a benchmark, we find that the international banking network became more prone to systemic risk after 1989 and peaked at the time of the Lehman

Brothers' collapse. Furthermore, it appears that the capacity of the international banking network to transmit contagion was not much less a year after the failure of Lehman Brothers.

It is important to understand that our results cannot be used to infer anything about the current riskiness of the system. The reason for this is that our contagion analysis only concerns the cross-sectional component of systemic risk and offers no insights as to changes in the average quality of banks' balance sheets over time. Contagion refers to the capacity to transmit stress. And, drawing an analogy to biological viruses, how contagious a disease is and how severe it is are separate issues.

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Appendix: Tables 1 and 2

Table 1a. Module members and prestige (selected quarters)

1985 Q1			1989 Q3			1989 Q4		
Members	Prestige of module	Contagion	Members	Prestige of module	Contagion	Members	Prestige of module	Contagion
US KY	25.3	9.0	GB JP US KY	62.6	9.7	US KY	21.6	7.3
GB	22.8	12.0	DE LU	8.0	3.7	GB	19.0	9.9
JP	9.2	4.2	FR	8.0	4.1	JP	18.1	8.2
FR	8.3	4.1	BE	4.6	2.3	CH LU	9.2	4.4
BE	7.7	3.4	IT	3.9	1.7	FR	7.9	3.9
DE LU	7.4	3.0	CH	2.6	1.3	BE	5.2	2.4
IT	4.4	2.0	NL	2.3	1.4	DE	4.7	2.8
CA	3.4	1.6	AT	1.4	0.7	IT	3.9	1.7
NL	2.7	1.7	CA	1.4	0.6	NL	2.8	1.5
CH	2.4	1.4	SE	1.3	0.5	CA	1.4	0.6
AT	1.5	0.7	DK	0.9	0.4	SE	1.2	0.5
ES	1.1	0.5	ES	0.8	0.4	AT	1.2	0.6
SE	0.9	0.4	FI	0.7	0.3	DK	1.0	0.5
DK	0.7	0.3	AU	0.6	0.3	ES	0.7	0.3
GR	0.6	0.3	GR	0.3	0.2	FI	0.5	0.2
FI	0.6	0.2	IE	0.3	0.1	AU	0.6	0.3
IE	0.3	0.2	PT	0.3	0.2	IE	0.3	0.2
AU	0.3	0.2				GR	0.3	0.2
PT	0.3	0.2				PT	0.2	0.17

Table 1b. Module members and prestige (selected quarters)

2000 Q1			2006 Q1			2007 Q1			2009 Q3		
Members	Prestige of module	Contagion	Members	Prestige of module	Contagion	Members	Prestige of module	Contagion	Members	Prestige of module	Contagion
CH GB	29.5	11.5	GB	22.8	10.8	GB	23.6	11.1	US KY	22.8	5.7
US KY	21.4	6.2	US KY	20.9	5.4	US KY	20.7	5.3	GB	21.0	10.1
DE LU	14.9	5.6	DE LU	13.3	5.8	DE LU	12.1	5.5	DE	8.6	4.9
FR	7.1	3.5	FR	8.7	4.5	FR	9.3	4.9	FR	8.5	4.6
JP	5.2	3.0	CH	5.0	2.8	NL	4.5	2.8	IT	4.6	2.1
NL	4.2	2.0	NL	4.9	2.4	CH	5.1	2.3	ES	4.5	1.9
IT	3.7	1.6	ES	3.8	1.6	IT	4.2	1.8	IE	4.4	2.0
BE	3.5	1.8	IT	3.7	1.6	ES	3.7	1.6	NL	4.2	2.0
IE	1.9	0.9	BE	3.3	1.7	IE	3.6	1.7	LU	3.6	1.8
ES	1.9	0.9	IE	3.3	1.5	BE	3.3	1.7	JP	3.5	2.0
CA	1.5	0.7	JP	3.1	1.8	DK FI SE	2.9	0.8	DK FI SE	3.1	0.8
FI SE	1.5	0.6	DK FI SE	2.7	0.7	JP	2.6	1.5	CH	3.0	1.7
AT	1.1	0.5	CA	1.2	0.5	CA	1.1	0.6	BE	2.6	1.4
PT	0.8	0.4	AT	1.2	0.6	AT	1.2	0.5	CA	1.5	0.7
DK	0.8	0.4	PT	1.2	0.5	PT	1.1	0.5	AT	1.3	0.6
AU	0.7	0.3	AU	0.7	0.3	AU	0.7	0.3	PT	1.1	0.5
GR	0.3	0.2	GR	0.3	0.2	GR	0.4	0.2	AU	1.0	0.5
									GR	0.6	0.3

Table 2a. Module members ordered by prestige (all quarters where there is change)

1985 Q1	1986 Q3	1987 Q3	1989 Q2	1989 Q3	1989 Q4	1992 Q1	1992 Q2	1992 Q3	1993 Q4	1997 Q1	1997 Q2
US KY	GB JP	GB JP US KY	GB JP	GB JP US KY	US KY	CH GB	US KY	CH GB	CH GB	CH GB	CH GB
GB	US KY	DE LU	US KY	DE LU	GB	US KY	GB	US KY	US KY	CA US KY	US KY
JP	DE LU	FR	DE LU	FR	JP	JP	JP	JP	DE LU	DE LU	DE LU
FR	FR	BE	FR	BE	CH LU	FR	CH LU	FR	JP	JP	JP
BE	BE	IT	BE	IT	FR	DE	FR	DE	FR	FR	FR
DE LU	IT	CH	IT	CH	BE	IT	DE	IT	IT	IT	IT
IT	CH	NL	CH	NL	DE	BE	IT	BE	BE	BE	BE
CA	NL	CA	NL	AT	IT	LU	BE	LU	NL	NL	NL
NL	CA	AT	CA	CA	NL	NL	NL	NL	ES	ES	ES
CH	AT	SE	AT	SE	CA	SE	SE	ES	CA	FI SE	CA
AT	DK	DK	SE	DK	SE	CA	CA	SE	AT	AT	AT
ES	SE	ES	DK	ES	AT	AT	ES	CA	SE	IE	IE
SE	ES	FI	ES	FI	DK	ES	AT	AT	DK	DK	SE
DK	FI	AU	FI	AU	ES	DK	DK	DK	IE	PT	DK
GR	AU	GR	AU	GR	FI	FI	FI	AU	AU	AU	PT
FI	GR	IE	IE	IE	AU	AU	AU	FI	PT	GR	AU
IE	IE	PT	GR	PT	IE	IE	IE	IE	FI		GR
AU	PT		PT		GR	GR	PT	PT	GR		FI
PT					PT	PT	GR	GR			

Table 2b. Module members ordered by prestige (all quarters where there is change)

1999 Q2	2000 Q3	2000 Q4	2001 Q1	2002 Q1	2003 Q1	2004 Q3	2005 Q1	2005 Q2	2007 Q4	2008 Q4	2009 Q1	2009 Q2
CH GB	CH GB	CH GB	CH GB	US KY	CH GB	GB	GB	GB	GB	US KY	US KY	US KY
US KY	US KY	US KY	US KY	GB	US KY	US KY	US KY	US KY	US KY	GB	GB	GB
DE LU	DE LU	DE LU	DE LU	DE LU	DE LU	DE LU	DE LU	DE LU	DE LU	DE	DE LU	DE
FR	FR	FR	FR	FR	FR	FR	BE NL	FR	FR	FR	FR	FR
JP	JP	JP	JP	CH	NL	CH	FR	CH	BE NL	IT	IT	IT
NL	NL	NL	NL	JP	JP	NL	CH	NL	CH	IE	ES	ES
IT	IT	IT	IT	NL	IT	JP	IT	IT	IT	ES	IE	IE
BE	BE	BE	BE	IT	BE	IT	JP	JP	ES	NL	NL	NL
ES	DK FI SE	IE	DK FI SE	BE	DK FI SE	BE	IE	IE	IE	LU	CH	LU
IE	IE	ES	IE	DK FI SE	IE	IE	ES	ES	DK FI SE	CH	DK FI SE	CH
FI SE	ES	CA	ES	IE	ES	ES	DK FI SE	BE	JP	JP	JP	JP
CA	CA	FI SE	CA	ES	CA	DK FI SE	PT	DK FI SE	CA	DK FI SE	BE	DK FI SE
AT	AT	AT	AT	CA	PT	PT	AT	PT	AT	BE	CA	BE
DK	PT	PT	PT	PT	AT	CA	CA	AT	PT	CA	AT	CA
PT	AU	DK	AU	AT	AU	AT	AU	CA	AU	AT	PT	AT
AU	GR	AU	GR	AU	GR	AU	GR	AU	GR	PT	AU	PT
GR		GR		GR		GR		GR	GR	AU	GR	AU
										GR		GR